Illustrations of the Logic of Science

CHARLES SANDERS PEIRCE
1839–1914

Illustrations of the Logic of Science

The 1877–78 text

Rendered into modern English with AI (Claude) and edited



FORAGE
forage.com

Illustrations of the Logic of Science · Charles Sanders Peirce (1839–1914)

Rendered into modern English with AI (Claude) and edited. Reading edition, Forage, September 2026. forage.com/peirce/logic-of-science

Original text: the six papers as first printed in *The Popular Science Monthly* (New York: D. Appleton and Company), vols. 12–13 (November 1877 – August 1878), via the validated transcription on Wikisource (en.wikisource.org/wiki/Popular_Science_Monthly/Volume_12/November_1877/Illustrations_of_the_Logic_of_Science_I) (text public domain; transcription CC BY-SA 4.0), with a few slips corrected against the page scans at the Internet Archive (archive.org/details/popularsciencemo12newy); Peirce's figures are omitted and his formulas set in plain text. Modern version: rewritten from Peirce's English with AI (Claude), then edited. Not reviewed by a human specialist. Free to reuse under CC BY-SA 4.0. The small numbers set in the text are paper and paragraph. Where the original uses a term now considered offensive, a neutral term is shown in brackets instead.

The original text is in the public domain. This translation is free to share and adapt under the Creative Commons Attribution-ShareAlike 4.0 International license (CC BY-SA 4.0): creativecommons.org/licenses/by-sa/4.0

Set in Source Serif 4.

CONTENTS

Introduction vii

The Fixation of Belief 1

THE FIXATION OF BELIEF I. 1

Reasoning and its guiding principles 4

Doubt and belief 7

Inquiry and the settlement of opinion 8

The method of tenacity 9

The method of authority 11

The a priori method 14

The method of science 16

How to Make Our Ideas Clear 21

HOW TO MAKE OUR IDEAS CLEAR I. 21

Belief as habit 25

Hardness, weight and force 31

Reality 36

Truth as the destined opinion 39

The Doctrine of Chances 45

THE DOCTRINE OF CHANCES I. 45

Probability as quantitative logic 48

What probability means 49

The single case 52

Logic and the unlimited community 54

Rules for calculating chances 57

The Probability of Induction 63

THE PROBABILITY OF INDUCTION I. 63

The conceptualistic view 65

Explicative and amplifiative reasoning 71

How often induction is right 76

How synthetic judgments are possible 79

The Order of Nature 83

THE ORDER OF NATURE I. 83

Is the world a chance-world? 85

Induction as sampling 91

Causes, space and time 95

Is the universe finite? 98

Deduction, Induction, and Hypothesis 103

DEDUCTION, INDUCTION, AND HYPOTHESIS I. 103

Making an hypothesis 105

The indirect moods 107

Rules for hypothesis 110

Induction and hypothesis in physics 113

Why the distinction matters 118

INTRODUCTION

How should we settle what to believe? In six papers written for *The Popular Science Monthly* in 1877 and 1878, the logician and scientist Charles Sanders Peirce set out to explain the logic of science to a general audience. The first, "The Fixation of Belief," describes inquiry as a struggle to escape the irritation of doubt, and weighs four ways of ending it—clinging to what we already think, obeying authority, following what seems agreeable to reason, and the method of science. The second, "How to Make Our Ideas Clear," gives the rule that William James would later make famous as pragmatism: our idea of anything is our idea of its sensible effects.

The remaining four papers turn to probability and induction: probability as a ratio in the long run, why reasoning about chances ties us to an unlimited community of inquirers, what we can and cannot infer from samples, whether the order of nature points to design, and the three kinds of inference—deduction, induction and hypothesis—illustrated with bags of beans.

The right-hand column is Peirce in lightly modernized English; the left is the text as first printed in the magazine.

The Fixation of Belief

Doubt and belief, and the four ways of settling opinion—tenacity, authority, the a priori method and the method of science.

THE FIXATION OF BELIEF I.

1.1 Few persons care to study logic, because everybody conceives himself to be proficient enough in the art of reasoning already. But I observe that this satisfaction is limited to one's own reasoning and does not extend to that of other people.

1.2 Our power of drawing inferences is the last of all our faculties to come into our full possession, for it is not so much a natural gift as a long and difficult art. The history of its practice would make a grand subject for a book. The medieval schoolmen, following the Romans, made logic the earliest of a boy's studies after grammar, because it was very easy. And so it was, as they understood it. Its fundamental principle, according to them, was that all knowledge rests either on authority or on reason, but that whatever is deduced by reason depends ultimately on a premise derived from authority.[1] Accordingly, as soon as a boy had mastered the procedure of the syllogism, his intellectual tool kit was considered complete.

1.3 To Roger Bacon, that remarkable mind who in the middle of the thirteenth century was almost a scientific man, the schoolmen's conception of reasoning seemed only an obstacle to truth. He saw that experience alone teaches anything. To us this

1 The schoolmen were the teachers of scholastic philosophy in the medieval universities.

proposition seems easy to understand, because a distinct conception of experience has been handed down to us from earlier generations; to him too it seemed perfectly clear, because its difficulties had not yet revealed themselves. Of all kinds of experience, the best, he thought, was inner illumination, which teaches many things about Nature that the external senses could never discover, such as the transubstantiation of bread.[2]

1.4 Four centuries later, the more famous Bacon, in the first book of his "Novum Organum," gave his clear account of experience as something that must be open to verification and reexamination. But, superior as Lord Bacon's conception is to earlier notions, a modern reader who is not in awe of his grand rhetoric is chiefly struck by how inadequate his view of scientific procedure is. That we have only to make some crude experiments, draw up summaries of the results on certain blank forms, go through these by rule, checking off everything disproved and setting down the alternatives, and that in this way physical science would be finished in a few years—what an idea! "He wrote on science like a Lord Chancellor," indeed.[3]

1.5 The early scientists—Copernicus, Tycho Brahe, Kepler, Galileo, and Gilbert—had methods more like those of their modern colleagues. Kepler set out to draw a curve through the positions of Mars;[4] and his greatest service to science was to impress on people's minds that this was the thing to be done if they wished to improve astronomy. They were not to content themselves with asking whether one system of epicycles was better than another; they were to sit down to the figures and find out what the curve really was.[5] He accomplished this by his incomparable energy and courage, blundering along in what seems to

2 Roger Bacon (c. 1219–c. 1292), English Franciscan philosopher who championed experiment and mathematics.

3 Francis Bacon (1561–1626), Lord Chancellor of England; the *Novum Organum* (1620) set out his inductive method. The quip about the Lord Chancellor is attributed to William Harvey.

4 PEIRCE'S NOTE. Not quite so, but as nearly so as can be told in a few words.

5 Epicycles: the small circles on circles by which pre-Keplerian astronomy modeled planetary motion.

us the most inconceivable way, from one irrational hypothesis to another, until, after trying twenty-two of them, he hit, through sheer exhaustion of his inventiveness, on the orbit that a mind well equipped with the weapons of modern logic would have tried almost at the outset.

1.6 In the same way, every work of science great enough to be remembered for a few generations offers some illustration of the defective state of the art of reasoning at the time it was written; and each major step in science has been a lesson in logic. So it was when Lavoisier and his contemporaries took up the study of chemistry.[6] The old chemist's maxim had been, "Read, read, read, work, pray, and read again." Lavoisier's method was not to read and pray; not to dream that some long and complicated chemical process would have a certain effect, put it into practice with dull patience, then, after its inevitable failure, dream that with some modification it would have another result, and end by publishing the last dream as a fact. His way was to carry his mind into his laboratory and to make his alembics and cucurbits instruments of thought. He gave a new conception of reasoning, as something to be done with one's eyes open, by manipulating real things instead of words and fancies.[7]

1.7 The Darwinian controversy is, in large part, a question of logic. Mr. Darwin proposed to apply the statistical method to biology. The same thing had been done in a very different branch of science, the theory of gases. Clausius and Maxwell could not say what the movements of any particular molecule of a gas would be on a certain hypothesis about the constitution of this class of bodies. Yet, by applying the doctrine of probabilities, they were able to predict that in the long run such and such a proportion of the molecules would, under given circumstances, acquire such and such velocities, that such and such a number of collisions would take place every second, and so on; and from these propositions they were able to deduce certain

6 Antoine Lavoisier (1743–1794), founder of modern quantitative chemistry.
7 Latin in the original: "Lege, lege, lege, labora, ora, et relege." Alembics and cucurbits are distilling vessels.

properties of gases, especially with regard to their heat-relations.[8] In the same way Darwin, while unable to say how variation and natural selection will operate in any individual case, demonstrates that in the long run they will adapt animals to their circumstances. Whether existing animal forms are due to such action, and what position the theory ought to take, is the subject of a discussion in which questions of fact and questions of logic are curiously interwoven.

REASONING AND ITS GUIDING PRINCIPLES

Validity is a matter of fact, and the habit behind an inference can be stated as a guiding principle.

1.8 The object of reasoning is to find out, from the consideration of what we already know, something else that we do not know. Consequently, reasoning is good if it is such as to give a true conclusion from true premises, and not otherwise. Thus the question of its validity is purely one of fact and not of thinking. If A is the premises and B the conclusion, the question is whether these facts are really so related that if A is, B is. If so, the inference is valid; if not, not. It is not in the least the question whether, when the mind accepts the premises, we feel an impulse to accept the conclusion also. It is true that by nature we do generally reason correctly. But that is an accident; the true conclusion would remain true if we had no impulse to accept it, and the false one would remain false even if we could not resist the tendency to believe it.

1.9 We are, no doubt, logical animals on the whole, but not perfectly so. Most of us, for example, are naturally more sanguine and hopeful than logic would justify. We seem to be so constituted that, when we have no facts to go on, we are happy and self-satisfied, so that the effect of experience is continually to shrink our hopes and aspirations. Yet a lifetime of applying this corrective does not usually root out our sanguine disposition.

8 Rudolf Clausius (1822–1888) and James Clerk Maxwell (1831–1879), founders of the kinetic theory of gases.

Where hope is not checked by any experience, our optimism is likely to be extravagant. Logicality in practical matters is the most useful quality an animal can possess, and might therefore result from the action of natural selection. But outside of these matters it is probably more advantageous for the animal to have its mind filled with pleasing and encouraging visions, regardless of their truth; and so, on unpractical subjects, natural selection might produce a fallacious tendency of thought.

1.10 What determines us, from given premises, to draw one inference rather than another is some habit of mind, whether constitutional or acquired. The habit is good or otherwise according as it does or does not produce true conclusions from true premises. And an inference is regarded as valid or not, not with special reference to the truth or falsity of its conclusion, but according as the habit that determines it is or is not such as to produce true conclusions in general. The particular habit of mind that governs this or that inference may be formulated in a proposition whose truth depends on the validity of the inferences the habit determines; such a formula is called a *guiding principle* of inference.[9] Suppose, for example, that we observe that a rotating copper disk quickly comes to rest when placed between the poles of a magnet, and we infer that this will happen with every copper disk. The guiding principle is that what is true of one piece of copper is true of another. Such a guiding principle would be much safer with regard to copper than with regard to many other substances—brass, for example.

1.11 A book might be written to point out all the most important of these guiding principles of reasoning. It would probably be, we must admit, of no use to a person whose thought is directed wholly to practical subjects and whose activity moves along thoroughly beaten paths. The problems that present themselves to such a mind are matters of routine, which he learned once and for all to handle when he learned his business. But let a man venture into an unfamiliar field, or one where his results are not continually checked by experience, and all history shows that

9 Peirce later calls this the "leading principle" of an inference.

the most vigorous intellect will often lose its bearings and waste its efforts in directions that bring it no nearer its goal, or even lead it entirely astray. He is like a ship on the open sea with no one on board who understands the rules of navigation. And in such a case some general study of the guiding principles of reasoning would certainly prove useful.

1.12 The subject could hardly be treated, however, without first being limited, since almost any fact may serve as a guiding principle. But it so happens that there is a division among facts such that one class contains all those that are absolutely essential as guiding principles, while the other contains all those that have any other interest as objects of research. The division is between the facts that are necessarily taken for granted in asking whether a certain conclusion follows from certain premises, and those that are not implied in that question. A moment's thought will show that a variety of facts are already assumed when the logical question is first asked. It is implied, for instance, that there are such states of mind as doubt and belief; that a passage from one to the other is possible, the object of thought remaining the same; and that this transition is subject to some rules by which all minds are alike bound. Since these are facts we must already know before we can have any clear conception of reasoning at all, inquiring into their truth or falsity can hardly be supposed to be of much further interest. On the other hand, it is easy to believe that the rules of reasoning deduced from the very idea of the process are the most essential ones; and, indeed, that so long as reasoning conforms to these it will at least not lead to false conclusions from true premises. In fact, what can be deduced from the assumptions involved in the logical question turns out to be more important than one might suppose, and this for reasons that are hard to show at the outset. The only one I shall mention here is that conceptions which are really products of logical reflection, without being readily seen to be so, mingle with our ordinary thoughts and are frequently the causes of great confusion. This is the case, for example, with the conception of quality. A quality as such is never an object of

observation. We can see that a thing is blue or green, but the quality of being blue and the quality of being green are not things we see; they are products of logical reflection. The truth is that common sense, or thought as it first rises above the level of the narrowly practical, is deeply imbued with that bad logical quality to which the epithet *metaphysical* is commonly applied; and nothing can clear it up but a rigorous course of logic.

DOUBT AND BELIEF

Doubt and belief feel different, and belief, unlike doubt, establishes a habit that will guide action.

1.13 We generally know when we wish to ask a question and when we wish to pronounce a judgment, for the sensation of doubting is unlike that of believing.

1.14 But this is not all that distinguishes doubt from belief. There is a practical difference. Our beliefs guide our desires and shape our actions. The Assassins, or followers of the Old Man of the Mountain, used to rush to their deaths at his slightest command, because they believed that obedience to him would ensure everlasting happiness. Had they doubted this, they would not have acted as they did. So it is with every belief, according to its degree. The feeling of believing is a more or less sure sign that some habit has been established in our nature which will determine our actions. Doubt never has such an effect.[10]

1.15 Nor must we overlook a third point of difference. Doubt is an uneasy and dissatisfied state from which we struggle to free ourselves and pass into the state of belief; whereas belief is a calm and satisfactory state that we do not wish to avoid, or to exchange for a belief in anything else.[11] On the contrary, we

10 The Assassins were a medieval Nizari Ismaili order; "the Old Man of the Mountain" was the name Crusader sources gave its leader.

11 PEIRCE'S NOTE. I am not speaking of secondary effects occasionally produced by the interference of other impulses.

cling tenaciously not merely to believing, but to believing just what we do believe.

1.16 Thus both doubt and belief have positive effects on us, though very different ones. Belief does not make us act at once, but puts us into such a condition that we shall behave in a certain way when the occasion arises. Doubt has not the least effect of this sort, but stimulates us to action until it is destroyed. This reminds us of the irritation of a nerve and the reflex action it produces; while for the analogue of belief in the nervous system we must look to what are called nervous associations—for example, to that habit of the nerves in consequence of which the smell of a peach will make the mouth water.

INQUIRY AND THE SETTLEMENT OF OPINION

The irritation of doubt drives inquiry, whose sole object is the settlement of opinion.

1.17 The irritation of doubt causes a struggle to attain a state of belief. I shall term this struggle *inquiry*, though it must be admitted that this is sometimes not a very apt name for it.

1.18 The irritation of doubt is the only immediate motive for the struggle to attain belief. It is certainly best for us that our beliefs should be such as may truly guide our actions so as to satisfy our desires; and this reflection will make us reject any belief that does not seem to have been formed so as to ensure this result. But it will do so only by creating a doubt in place of that belief. With the doubt, therefore, the struggle begins, and with the end of doubt it ends. Hence the sole object of inquiry is the settlement of opinion. We may imagine that this is not enough for us, and that we seek not merely an opinion but a true opinion. But put this notion to the test and it proves groundless; for as soon as a firm belief is reached we are entirely satisfied, whether the belief is true or false. And it is clear that nothing outside the sphere of our knowledge can be our object, for nothing that does not affect the mind can be the motive for a mental effort. The most that can be maintained is that we seek a belief

that we shall *think* to be true. But we think each one of our beliefs to be true, and indeed it is mere tautology to say so.

1.19 That the settlement of opinion is the sole end of inquiry is a very important proposition. It sweeps away at once various vague and erroneous conceptions of proof. A few of these may be noted here.

1.20 1. Some philosophers have imagined that to start an inquiry it was only necessary to utter a question or write it down on paper, and have even recommended that we begin our studies by questioning everything! But merely putting a proposition in the form of a question does not stimulate the mind to any struggle after belief. There must be a real and living doubt, and without this all discussion is idle.[12]

1.21 2. It is a very common idea that a demonstration must rest on some ultimate and absolutely indubitable propositions. According to one school these are first principles of a general nature; according to another, they are first sensations. But in fact, for an inquiry to have that completely satisfactory result called demonstration, it has only to start with propositions perfectly free from all actual doubt. If the premises are not in fact doubted at all, they cannot be more satisfactory than they are.

1.22 3. Some people seem to love to argue a point after the whole world is fully convinced of it. But no further advance can be made. When doubt ceases, mental action on the subject comes to an end; and if it did go on, it would be without a purpose.

THE METHOD OF TENACITY

One may fix belief by clinging to any chosen answer and shunning whatever might disturb it.

1.23 If the settlement of opinion is the sole object of inquiry, and if belief is of the nature of a habit, why should we not attain the desired end by taking any answer to a question that we may fancy, and constantly repeating it to ourselves, dwelling on

12 The allusion is chiefly to Descartes's method of universal doubt.

everything that may conduce to that belief, and learning to turn with contempt and hatred from anything that might disturb it? This simple and direct method is really followed by many people. I remember once being begged not to read a certain newspaper in case it might change my opinion on free trade. "Lest I might be entrapped by its fallacies and misstatements," was how it was put. "You are not," my friend said, "a special student of political economy. You might therefore easily be deceived by fallacious arguments on the subject. So, if you read this paper, you might be led to believe in protection. But you admit that free trade is the true doctrine, and you do not wish to believe what is not true." I have often known this system to be deliberately adopted. Still more often, the instinctive dislike of an undecided state of mind, exaggerated into a vague dread of doubt, makes people cling spasmodically to the views they already hold. The man feels that if he only holds to his belief without wavering, it will be entirely satisfactory.[13] Nor can it be denied that a steady and immovable faith yields great peace of mind. It may, indeed, give rise to inconveniences—as if a man should resolutely go on believing that fire would not burn him, or that he would be eternally damned if he received his *ingesta* otherwise than through a stomach-pump. But then the man who adopts this method will not allow that its inconveniences outweigh its advantages. He will say, "I hold steadfastly to the truth, and the truth is always wholesome." And in many cases it may very well be that the pleasure he derives from his calm faith outweighs any inconveniences resulting from its deceptive character.[14] Thus, if it is true that death is annihilation, then the man who believes he will certainly go straight to heaven when he dies, provided he has fulfilled certain simple observances in this life, has a cheap pleasure that will not be followed by the least disappointment. A similar consideration seems to carry weight with many people on religious topics, for we often hear it said, "Oh, I could not believe so-and-so, because I should be wretched

13 "Protection": protective tariffs, the policy opposed to free trade.
14 *Ingesta*: Latin for what is taken into the body as food and drink.

if I did." When an ostrich buries its head in the sand as danger approaches, it very likely takes the happiest course. It hides the danger, and then calmly says there is no danger; and if it feels perfectly sure there is none, why should it raise its head to look? A man may go through life systematically keeping out of view everything that might change his opinions, and if he only succeeds—basing his method, as he does, on two fundamental psychological laws—I do not see what can be said against his doing so. It would be an egotistical impertinence to object that his procedure is irrational, for that only amounts to saying that his method of settling belief is not ours. He does not set out to be rational, and indeed will often speak with scorn of man's weak and illusory reason. So let him think as he pleases.

1.24 But this method of fixing belief, which may be called the method of tenacity, will be unable to hold its ground in practice. The social impulse is against it. The man who adopts it will find that other people think differently from him, and in some saner moment it will be apt to occur to him that their opinions are quite as good as his own, and this will shake his confidence in his belief. This conception, that another man's thought or feeling may be equivalent to one's own, is a distinctly new step, and a highly important one. It arises from an impulse too strong in man to be suppressed without danger of destroying the human species. Unless we make ourselves hermits, we shall necessarily influence each other's opinions; so the problem becomes how to fix belief, not in the individual merely, but in the community.

THE METHOD OF AUTHORITY

The state fixes belief by teaching approved doctrines and suppressing dissent, with great success and great cruelty.

1.25 Let the will of the state act, then, instead of that of the individual. Let an institution be created whose object is to keep correct doctrines before the attention of the people, to repeat them perpetually, and to teach them to the young, while having at the same time the power to prevent contrary doctrines from being

taught, advocated, or expressed. Let all possible causes of a change of mind be removed from people's awareness. Let them be kept ignorant, in case they learn of some reason to think otherwise than they do. Let their passions be enlisted, so that they regard private and unusual opinions with hatred and horror. Then let all who reject the established belief be terrified into silence. Let the people turn out and tar and feather such men, or let inquisitions be made into the way suspected persons think, and when they are found guilty of forbidden beliefs, let them be subjected to some exemplary punishment. When complete agreement could not otherwise be reached, a general massacre of all who have not thought in a certain way has proved a very effective means of settling opinion in a country. If the power to do this is lacking, let a list of opinions be drawn up to which no one with the least independence of thought can assent, and let the faithful be required to accept all these propositions, in order to segregate them as radically as possible from the influence of the rest of the world.

1.26 This method has, from the earliest times, been one of the chief means of upholding correct theological and political doctrines and of preserving their universal or catholic character. In Rome especially it has been practiced from the days of Numa Pompilius to those of Pius IX. This is the most perfect example in history; but wherever there is a priesthood—and no religion has been without one—this method has been used to a greater or lesser extent. Wherever there is an aristocracy, or a guild, or any association of a class of people whose interests depend, or are supposed to depend, on certain propositions, some traces of this natural product of social feeling will inevitably be found.[15] Cruelties always accompany this system; and when it is consistently carried out, they become, in the eyes of any rational person, atrocities of the most horrible kind. Nor should this be surprising, for the officer of a society does not feel justified in sur-

15 Numa Pompilius, legendary second king of Rome, credited with founding its religious institutions; Pius IX (in the original, "Pius Nonus") was pope from 1846 to 1878.

rendering that society's interests for the sake of mercy, as he might his own private interests. It is natural, therefore, that sympathy and fellowship should in this way produce a most ruthless power.

1.27 In judging this method of fixing belief, which may be called the method of authority, we must first of all grant its immeasurable mental and moral superiority to the method of tenacity. Its success is proportionately greater; and in fact it has again and again produced the most majestic results. The mere structures of stone it has caused to be built—in Siam, for example, in Egypt, and in Europe—many of them have a sublimity scarcely surpassed by the greatest works of Nature. And, apart from the geological epochs, there are no periods of time as vast as those measured by some of these organized faiths. If we scrutinize the matter closely, we shall find that not one of their creeds has remained always the same; yet the change is so slow as to be imperceptible within one person's life, so that individual belief remains, for all practical purposes, fixed. For the mass of mankind, then, there is perhaps no better method than this. If it is their highest impulse to be intellectual slaves, then slaves they ought to remain.

1.28 But no institution can undertake to regulate opinions on every subject. Only the most important ones can be attended to, and on the rest people's minds must be left to the action of natural causes. This imperfection will be no source of weakness so long as people are at such a level of culture that one opinion does not influence another—that is, so long as they cannot put two and two together. But even in the most priest-ridden states some individuals will be found who have risen above that condition. These people possess a wider sort of social feeling. They see that people in other countries and other ages have held very different doctrines from those they themselves were brought up to believe; and they cannot help seeing that it is the mere accident of their having been taught as they were, and of their having been surrounded by the manners and associations they were, that has caused them to believe as they do and not very

differently. And their candor cannot resist the reflection that there is no reason to value their own views more highly than those of other nations and other centuries; and this gives rise to doubts in their minds.

THE A PRIORI METHOD

Opinion is left to settle on what seems agreeable to reason, which makes inquiry a matter of taste.

1.29 They will further perceive that doubts like these must exist in their minds with regard to every belief that seems to be determined by the caprice either of themselves or of those who originated the popular opinions. The willful adherence to a belief, and the arbitrary forcing of it on others, must therefore both be given up, and a new method of settling opinions must be adopted—one that will not only produce an impulse to believe, but will also decide which proposition is to be believed. Let the action of natural preferences be unimpeded, then, and under their influence let people, conversing together and looking at matters in different lights, gradually develop beliefs in harmony with natural causes. This method resembles the one by which conceptions of art have been brought to maturity. Its most perfect example is to be found in the history of metaphysical philosophy. Systems of this sort have not usually rested on any observed facts, at least not to any great degree. They have been adopted chiefly because their fundamental propositions seemed "agreeable to reason." This is an apt expression: it does not mean what agrees with experience, but what we find ourselves inclined to believe. Plato, for example, finds it agreeable to reason that the distances of the celestial spheres from one another should be proportional to the different lengths of strings that produce harmonious chords. Many philosophers have been led to their main conclusions by considerations like this. But this is the lowest and least developed form the method takes, for clearly another man might find Kepler's theory—that the celestial spheres are proportional to the inscribed and cir-

cumscribed spheres of the different regular solids—more agreeable to *his* reason. But the clash of opinions will soon lead people to rely on preferences of a far more universal nature. Take, for example, the doctrine that man acts only selfishly—that is, from the consideration that acting in one way will give him more pleasure than acting in another.[16] This rests on no fact in the world, but it has been widely accepted as the only reasonable theory.

1.30 This method is far more intellectual and respectable, from the point of view of reason, than either of the others we have considered. But its failure has been the most obvious. It makes inquiry something like the development of taste; but taste, unfortunately, is always more or less a matter of fashion. Accordingly, metaphysicians have never come to any fixed agreement; instead, the pendulum has swung back and forth between a more material and a more spiritual philosophy, from the earliest times to the latest. And so from this method, which has been called the *a priori* method, we are driven, in Lord Bacon's phrase, to a true induction. We have examined this *a priori* method as something that promised to free our opinions from their accidental and capricious element. But development, while it is a process that eliminates the effect of some chance circumstances, only magnifies that of others. This method, therefore, does not differ in any very essential way from that of authority. The government may not have lifted a finger to influence my convictions; I may have been left outwardly quite free to choose, let us say, between monogamy and polygamy, and, appealing to my conscience alone, I may have concluded that the latter practice is in itself licentious. But when I come to see that the chief obstacle to the spread of Christianity among a people as highly cultured as the Hindus has been a conviction that our way of treating women is immoral, I cannot help seeing that, even though governments do not interfere, sentiments in their development will be very largely determined by accidental causes. Now, there are some people—and I must suppose my

16 Kepler's scheme appears in his *Mysterium Cosmographicum* (1596).

reader is among them—who, when they see that any belief of theirs is determined by some circumstance extraneous to the facts, will from that moment not merely admit in words that the belief is doubtful, but will experience a real doubt of it, so that it ceases to be a belief.

THE METHOD OF SCIENCE

Belief is to be fixed by real things, which lead every inquirer to the same conclusion.

1.31 To satisfy our doubts, therefore, a method must be found by which our beliefs may be caused by nothing human, but by some external permanency—by something on which our thinking has no effect. Some mystics imagine they have such a method in a private inspiration from on high. But that is only a form of the method of tenacity, in which the conception of truth as something public has not yet developed. Our external permanency would not be external, in our sense, if its influence were restricted to one individual. It must be something that affects, or might affect, every person. And though these effects are necessarily as various as individual conditions are, yet the method must be such that the ultimate conclusion of every person will be the same. Such is the method of science. Its fundamental hypothesis, restated in more familiar language, is this: There are real things, whose characters are entirely independent of our opinions about them; those realities affect our senses according to regular laws; and, though our sensations are as different as our relations to the objects, yet by taking advantage of the laws of perception we can ascertain by reasoning how things really are, and anyone, if he has sufficient experience and reasons enough about it, will be led to the one true conclusion. The new conception involved here is that of reality. It may be asked how I know that there are any realities. If this hypothesis is the sole support of my method of inquiry, my method of inquiry must not be used to support my hypothesis. The reply is this: 1. If investigation cannot be regarded as proving that there

are real things, it at least does not lead to a contrary conclusion; the method and the conception on which it is based remain always in harmony. No doubts about the method, therefore, necessarily arise from its practice, as is the case with all the others. 2. The feeling that gives rise to any method of fixing belief is a dissatisfaction at two incompatible propositions. But here already is a vague admission that there is some *one* thing to which a proposition should conform. Nobody, therefore, can really doubt that there are realities; or, if he did, doubt would not be a source of dissatisfaction. The hypothesis, therefore, is one that every mind admits. So the social impulse does not cause me to doubt it. 3. Everybody uses the scientific method about a great many things, and stops using it only when he does not know how to apply it. 4. Experience of the method has not led me to doubt it; on the contrary, scientific investigation has had the most wonderful triumphs in the way of settling opinion. These considerations explain why I do not doubt the method or the hypothesis it presupposes; and since I have no doubt, and do not believe that anyone else I could influence has any, it would be the merest babble for me to say more about it. If there is anyone with a living doubt on the subject, let him consider it.

1.32 To describe the method of scientific investigation is the object of this series of papers. For now I have room only to note some points of contrast between it and other methods of fixing belief.

1.33 This is the only one of the four methods that presents any distinction between a right and a wrong way. If I adopt the method of tenacity and shut myself off from all influences, whatever I think necessary to doing this is necessary according to that method. So with the method of authority: the state may try to put down heresy by means that, from a scientific point of view, seem very poorly suited to its purposes; but the only test *on that method* is what the state thinks, so that it cannot pursue the method wrongly. So with the *a priori* method. Its very essence is to think as one is inclined to think. All metaphysicians will be sure to do that, however inclined they may be to

judge one another perversely wrong. The Hegelian system recognizes every natural tendency of thought as logical, even though it is certain to be abolished by counter-tendencies. Hegel thinks there is a regular system in the succession of these tendencies, in consequence of which, after drifting one way and the other for a long time, opinion will at last go right. And it is true that metaphysicians do get the right ideas in the end: Hegel's system of Nature represents fairly well the science of his day; and one may be sure that whatever scientific investigation has put beyond doubt will soon receive *a priori* demonstration on the part of the metaphysicians. But with the scientific method the case is different. I may start from known and observed facts in order to proceed to the unknown, and yet the rules I follow in doing so may not be such as investigation would approve. The test of whether I am truly following the method is not an immediate appeal to my feelings and purposes; on the contrary, it itself involves applying the method. That is why bad reasoning as well as good reasoning is possible; and this fact is the foundation of the practical side of logic.

1.34 We should not suppose that the first three methods of settling opinion have no advantage at all over the scientific method. On the contrary, each has some convenience peculiar to itself. The *a priori* method is distinguished by its comfortable conclusions. It is the nature of the process to adopt whatever belief we are inclined to, and there are certain flatteries to human vanity which we all believe by nature, until some rough facts wake us from our pleasant dream. The method of authority will always govern the mass of mankind. Those who wield the various forms of organized force in the state will never be convinced that dangerous reasoning ought not to be suppressed in some way. If freedom of speech is to be released from the cruder forms of constraint, then uniformity of opinion will be secured by a moral terrorism which respectable society will wholeheartedly approve. Following the method of authority is the path of peace. Certain nonconformities are permitted; certain others, considered unsafe, are forbidden. These differ from

country to country and from age to age. But wherever you are, let it be known that you seriously hold a taboo belief, and you may be perfectly sure of being treated with a cruelty less brutal, but more refined, than hunting you down like a wolf. So the greatest intellectual benefactors of mankind have never dared, and do not dare now, to utter the whole of their thought; and so a shade of *prima facie* doubt is cast on every proposition considered essential to the security of society. Oddly enough, the persecution does not all come from outside. A man torments himself, and is often most distressed to find himself believing propositions he was brought up to regard with aversion. The peaceful and sympathetic man will therefore find it hard to resist the temptation to submit his opinions to authority. But most of all I admire the method of tenacity for its strength, simplicity, and directness.[17] The men who follow it are distinguished by their decisiveness of character, which comes very easily with such a mental rule. They do not waste time trying to make up their minds what they want. Instead they fasten like lightning on whatever alternative comes first and hold to it to the end, whatever happens, without an instant's wavering. This is one of the splendid qualities that generally go with brilliant but short-lived success. It is impossible not to envy the man who can dismiss reason, even though we know how things must turn out for him in the end.

1.35 These are the advantages that the other methods of settling opinion have over scientific investigation. A man should consider them well. Then he should consider that, after all, he wants his opinions to coincide with the fact, and that there is no reason why the results of these three methods should do so. Bringing about this coincidence is the special privilege of the method of science. On such considerations he has to make his choice—a choice that is far more than the adoption of an intellectual opinion. It is one of the ruling decisions of his life, and once it is made he is bound to stand by it. The force of habit will

17 *Prima facie*: Latin, "at first sight"; a presumptive doubt that holds until examined.

sometimes make a man hold on to old beliefs after he is in a position to see that they have no sound basis. But reflection on the state of the case will overcome these habits, and he ought to give reflection its full weight. People sometimes shrink from doing this, because they have an idea that some beliefs are wholesome even though they cannot help feeling that those beliefs rest on nothing. Such people should imagine a case that is analogous to their own but different from it. Let them ask themselves what they would say to a former Muslim who hesitated to give up his old notions about the relations of the sexes, or to a former Catholic who still shrank from reading the Bible. Would they not say that these people ought to consider the matter fully, understand the new doctrine clearly, and then embrace it in its entirety? But above all, let it be considered that what is more wholesome than any particular belief is integrity of belief. To avoid looking into the support of a belief for fear that it may turn out to be rotten is quite as immoral as it is disadvantageous. Suppose a person admits that there is such a thing as truth, distinguished from falsehood simply by this: that if we act on it, it will carry us to the point we aim at and not lead us astray. If, though convinced of this, he does not dare to know the truth and tries to avoid it, he is in a sorry state of mind indeed.

1.36 Yes, the other methods do have their merits. A clear logical conscience does cost something—just as any virtue, just as everything we cherish, costs us dearly. But we should not wish it otherwise. The spirit of a man's logical method should be loved and revered as his bride, whom he has chosen from all the world. He need not despise the others; on the contrary, he may honor them deeply, and in doing so he only honors her the more. But she is the one he has chosen, and he knows he was right to choose her. And having chosen her, he will work and fight for her. He will not complain that there are blows to take, but will hope there may be as many and as hard to give. He will strive to be the worthy knight and champion of her from the blaze of whose splendor he draws his inspiration and his courage.

How to Make Our Ideas Clear

Beyond clearness and distinctness: the pragmatic maxim, and what it makes of hardness, weight, force and reality.

HOW TO MAKE OUR IDEAS CLEAR I.

2.1 Anyone who has looked into an ordinary modern treatise on logic will doubtless remember the two distinctions between *clear* and *obscure* conceptions, and between *distinct* and *confused* conceptions. They have lain in the books for nearly two centuries now, unimproved and unmodified, and logicians generally count them among the gems of their doctrine.

2.2 A clear idea is defined as one that is grasped in such a way that it will be recognized wherever it is met with, and that no other idea will be mistaken for it. If it falls short of this clearness, it is said to be obscure.

2.3 This is a rather neat bit of philosophical terminology. Still, since it was clearness they were defining, I wish the logicians had made their definition a little plainer. Never to fail to recognize an idea, and under no circumstances to mistake another for it, however obscure the form in which it appears, would indeed imply a prodigious force and clearness of intellect such as is seldom met with in this world. On the other hand, merely to know an idea well enough to have become familiar with it, and to have lost all hesitation in recognizing it in ordinary cases, hardly seems to deserve the name of clearness of apprehension. After all, it amounts only to a subjective feeling of mastery, which may be entirely mistaken. I take it, however, that when the logicians speak of "clearness," they mean nothing more than this sort of familiarity with an idea, since they regard the quality as

only a small merit, which needs to be supplemented by another, which they call *distinctness.*

2.4 A distinct idea is defined as one that contains nothing that is not clear. This is technical language: by the *contents* of an idea logicians mean whatever is contained in its definition. So, according to them, an idea is *distinctly* apprehended when we can give a precise definition of it in abstract terms. Here the professional logicians leave the subject. I would not have troubled the reader with what they have to say, if it were not such a striking example of how they have been slumbering through ages of intellectual activity, listlessly ignoring the machinery of modern thought and never dreaming of applying its lessons to the improvement of logic. It is easy to show that the doctrine that familiar use and abstract distinctness make up the perfection of apprehension has its only proper place in philosophies that have long been extinct. It is now time to formulate the method of attaining a more perfect clearness of thought, of the kind we see and admire in the thinkers of our own time.

2.5 When Descartes set about reconstructing philosophy, his first step was to permit skepticism (in theory) and to discard the practice of the schoolmen, who looked to authority as the ultimate source of truth. That done, he looked for a more natural fountain of true principles, and claimed to find it in the human mind. He thus passed, in the most direct way, from the method of authority to that of apriority, as described in my first paper. Self-consciousness was to furnish us with our fundamental truths and to decide what was agreeable to reason. But since evidently not all ideas are true, he was led to note, as the first condition of infallibility, that they must be clear.[1] The distinction between an idea *seeming* clear and really being clear never occurred to him. Trusting introspection as he did, even for knowledge of external things, why should he question its testimony about the contents of our own minds? But then, I suppose, seeing men who seemed quite clear and positive holding opposite opinions on fundamental principles, he was further led to

1 René Descartes (1596–1650), French philosopher.

say that clearness of ideas is not enough: they also need to be distinct, that is, to have nothing unclear about them. What he probably meant by this (for he did not explain himself precisely) was that they must withstand the test of dialectical examination: they must not only seem clear at the outset, but discussion must never be able to bring to light points of obscurity connected with them.

2.6 Such was Descartes' distinction, and one can see that it was exactly on the level of his philosophy. Leibniz developed it somewhat. This great and singular genius was as remarkable for what he failed to see as for what he saw. It was perfectly apparent to him that a piece of machinery could not do work perpetually without being fed with power in some form. Yet he did not understand that the machinery of the mind can only transform knowledge, never originate it, unless it is fed with facts of observation.[2] He thus missed the most essential point of the Cartesian philosophy, which is that accepting propositions that seem perfectly evident to us is something we cannot help doing, whether it is logical or illogical. Instead of seeing the matter this way, he tried to reduce the first principles of science to formulas that cannot be denied without self-contradiction, and he was apparently unaware of the great difference between his position and Descartes'. So he went back to the old formalities of logic, and above all, abstract definitions played a great part in his philosophy. It was quite natural, therefore, that when he observed that Descartes' method suffered from the difficulty that we may seem to ourselves to have clear apprehensions of ideas that are in truth very hazy, no better remedy occurred to him than to require an abstract definition of every important term. Accordingly, in adopting the distinction of *clear* and *distinct* notions, he described the second quality as the clear apprehension of everything contained in the definition; and the books have copied his words ever since. There is no danger that his fanciful scheme will ever again be overvalued. Nothing new can ever be

2 Gottfried Wilhelm Leibniz (1646–1716), German philosopher and mathematician (Peirce spells him "Leibnitz").

learned by analyzing definitions. Still, our existing beliefs can be put in order by this process, and order is an essential element of intellectual economy, as of every other kind. It may be granted, then, that the books are right to make familiarity with a notion the first step toward clearness of apprehension, and defining it the second. But in omitting all mention of any higher lucidity of thought, they simply mirror a philosophy that was exploded a hundred years ago. That much-admired "ornament of logic"—the doctrine of clearness and distinctness—may be pretty enough; but it is high time to consign the antique *bijou* to our cabinet of curiosities, and to wear something better suited to modern uses.[3]

2.7 The very first lesson we have a right to demand that logic teach us is how to make our ideas clear. It is a most important one, belittled only by minds that stand in need of it. To know what we think, to be masters of our own meaning, will make a solid foundation for great and weighty thought. It is most easily learned by those whose ideas are meager and restricted, and they are far happier than those who wallow helplessly in a rich mud of conceptions. A nation, it is true, may over the course of generations overcome the disadvantage of an excessive wealth of language and its natural companion, a vast, unfathomable deep of ideas. We can see such a nation in history, slowly perfecting its literary forms, at length shedding its metaphysics, and, by virtue of the tireless patience that is often its compensation, attaining great excellence in every branch of mental achievement. The page of history has not yet been unrolled that will tell us whether such a people will or will not prevail in the long run over one whose ideas (like the words of its language) are few, but which has a wonderful mastery over those it has. For an individual, however, there can be no question that a few clear ideas are worth more than many confused ones. A young man could hardly be persuaded to sacrifice the greater part of his thoughts to save the rest; and the muddled head is the least likely to see the need for such a sacrifice. We can usually only

3 *Bijou*: French, "jewel" or "trinket."

pity him, as a person with a congenital defect. Time will help him, but intellectual maturity with regard to clearness comes rather late. This is an unfortunate arrangement of Nature, since clearness is of less use to a man settled in life, whose errors have largely had their effect, than it would be to one whose path lies before him. It is terrible to see how a single unclear idea, a single meaningless formula, lurking in a young man's head, will sometimes act like a blockage of inert matter in an artery, hindering the nourishment of the brain and condemning its victim to waste away in the fullness of his intellectual vigor and in the midst of intellectual plenty. Many a man has cherished for years, as his hobby, some vague shadow of an idea, too meaningless to be positively false. He has nevertheless loved it passionately, made it his companion by day and by night, and given it his strength and his life, leaving all other occupations for its sake. In short, he has lived with it and for it, until it has become, as it were, flesh of his flesh and bone of his bone. And then he has woken up some bright morning to find it gone, clean vanished away like the beautiful Melusina of the fable, and the essence of his life gone with it.[4] I have known such a man myself. And who can tell how many histories of circle-squarers, metaphysicians, astrologers, and what not, might be told in the old German story?

BELIEF AS HABIT

Thought is roused by doubt and ends in belief, a rule of action whose meaning lies in habits.

2.8 The principles set out in the first of these papers lead at once to a method of reaching a clearness of thought of a far higher grade than the "distinctness" of the logicians. We found there that the action of thought is excited by the irritation of doubt and ceases when belief is attained, so that the production of belief is the sole function of thought. All these words, however,

4 Melusina: a water-fairy of European legend who married a mortal and vanished forever when he broke his promise not to look at her on a certain day.

are too strong for my purpose. It is as if I had described the phenomena as they appear under a mental microscope. Doubt and Belief, as the words are commonly used, relate to religious or other serious discussions. But here I use them to designate the raising of any question, no matter how small or how great, and its resolution. If, for instance, I pull out my purse in a horse-car and find a five-cent nickel and five coppers, I decide, while my hand is going to the purse, how I will pay my fare. To call such a question Doubt, and my decision Belief, is certainly to use words very out of proportion to the occasion. To speak of such a doubt as causing an irritation that needs to be appeased suggests a temperament uncomfortable to the verge of insanity.[5] Yet, looking at the matter closely, it must be admitted that if there is the least hesitation about whether I shall pay the five coppers or the nickel (as there is sure to be, unless I act from some habit previously formed in the matter), then, though irritation is too strong a word, I am stirred to whatever small mental activity may be needed to decide how I shall act. Most often doubts arise from some indecision, however momentary, in our action. Sometimes this is not so. For example, I have to wait in a railway station, and to pass the time I read the advertisements on the walls. I compare the advantages of different trains and different routes that I never expect to take, merely pretending to be in a state of hesitation, because I am bored with having nothing to trouble me. Feigned hesitation, whether feigned for mere amusement or with a lofty purpose, plays a great part in the production of scientific inquiry. However the doubt may originate, it stimulates the mind to an activity that may be slight or energetic, calm or turbulent. Images pass rapidly through consciousness, one incessantly melting into another, until at last, when all is over—it may be in a fraction of a second, in an hour, or after long years—we find ourselves decided as to how we should act under circumstances like those that caused our hesitation. In other words, we have attained belief.

2.9 In this process we observe two sorts of elements of consciousness, and the distinction between them is best made clear by an illustration. In a piece of music there are the separate notes, and there is the melody. A single tone may be held for an hour or a day, and it exists as perfectly in each second of that time as in the whole taken together. So, as long as it is sounding, it could be present to a sense from which everything in the past was as completely absent as the future itself. But it is different with the melody. Its performance takes a certain time, and during the portions of that time only portions of it are played. It consists in an orderliness in the succession of sounds that strike the ear at different times; and to perceive it there must be some continuity of consciousness that makes the events of a stretch of time present to us. We certainly perceive the melody only by hearing the separate notes; yet we cannot be said to hear it directly, for we hear only what is present at the instant, and an orderliness of succession cannot exist in an instant. These two sorts of objects, what we are *immediately* conscious of and what we are *mediately* conscious of, are found in all consciousness. Some elements (the sensations) are completely present at every instant for as long as they last. Others (like thought) are actions having a beginning, middle, and end, and they consist in a congruence in the succession of sensations that flow through the mind. They cannot be immediately present to us, but must cover some portion of the past or future. Thought is a thread of melody running through the succession of our sensations.

2.10 We may add that, just as a piece of music may be written in parts, each part having its own melody, so various systems of relations of succession exist together among the same sensations. These different systems are distinguished by having different motives, ideas, or functions. Thought is only one such system, for its sole motive, idea, and function is to produce belief, and whatever does not concern that purpose belongs to some other system of relations. The action of thinking may incidentally have other results. It may serve to amuse us, for example;

5 A horse-car was a horse-drawn streetcar; “coppers” are one-cent pieces.

and among *dilettanti* it is not rare to find people who have so perverted thought to the purposes of pleasure that it seems to annoy them to think that the questions on which they delight to exercise it might ever be finally settled. A positive discovery that takes a favorite subject out of the arena of literary debate is met with ill-concealed dislike. This disposition is the very debauchery of thought. But the soul and meaning of thought, abstracted from the other elements that accompany it, though it may be deliberately thwarted, can never be made to direct itself toward anything but the production of belief. Thought in action has for its only possible motive the attainment of thought at rest; and whatever does not refer to belief is no part of the thought itself.

2.11 What, then, is belief? It is the half cadence that closes a musical phrase in the symphony of our intellectual life. We have seen that it has just three properties. First, it is something we are aware of; second, it appeases the irritation of doubt; and third, it involves the establishment in our nature of a rule of action, or, for short, a *habit*. Since it appeases the irritation of doubt, which is the motive for thinking, thought relaxes and comes to rest for a moment when belief is reached.[6] But since belief is a rule for action, and applying it involves further doubt and further thought, it is at once a stopping place and a new starting place for thought. That is why I have allowed myself to call it thought at rest, although thought is essentially an action. The *final* upshot of thinking is the exercise of the will, and of this thought no longer forms a part. But belief is only a stage of mental action, an effect on our nature due to thought, which will influence future thinking.

2.12 The essence of belief is the establishment of a habit, and different beliefs are distinguished by the different modes of action to which they give rise. If beliefs do not differ in this respect—if they appease the same doubt by producing the same rule of action—then no mere differences in the way we are con-

6 A half cadence (Peirce's "demi-cadence") is a musical close that pauses without full resolution, leaving the music to continue.

scious of them can make them different beliefs, any more than playing a tune in different keys is playing different tunes. Imaginary distinctions are often drawn between beliefs that differ only in how they are expressed; the wrangling that follows is real enough, however. To believe that any objects are arranged as in Fig. 1, and to believe that they are arranged as in Fig.[7] 2, are one and the same belief; yet it is conceivable that a man might assert the one proposition and deny the other. Such false distinctions do as much harm as confusing beliefs that really are different, and they are among the pitfalls we should constantly beware of, especially on metaphysical ground. One peculiar deception of this sort, which occurs often, is to mistake the sensation produced by our own unclearness of thought for a character of the object we are thinking about. Instead of seeing that the obscurity is purely subjective, we imagine that we are contemplating a quality of the object that is essentially mysterious. And if our conception is later presented to us in a clear form, we do not recognize it as the same, because the feeling of unintelligibility is missing. As long as this deception lasts, it obviously puts an impassable barrier in the way of lucid thinking. So it is equally in the interest of the opponents of rational thought to perpetuate it, and of its adherents to guard against it.

2.13 Another such deception is to mistake a mere difference in the grammatical construction of two words for a distinction between the ideas they express. In this pedantic age, when the general mob of writers attend so much more to words than to things, this error is common enough. When I said just now that thought is an *action*, and that it consists in a *relation*, there was no inconsistency in what I said, but only a grammatical vagueness—even though a person performs an action but not a relation, which can only be the result of an action.

2.14 We shall be perfectly safe from all these sophisms so long as we reflect that the whole function of thought is to produce habits of action, and that whatever is connected with a thought but irrelevant to its purpose is an accretion to it, but no part of

7 Peirce's figures are not reproduced here.

it. If there is a unity among our sensations that has no reference to how we shall act on a given occasion, as when we listen to a piece of music, why, we do not call that thinking. To develop a thought's meaning, then, we have simply to determine what habits it produces, for what a thing means is simply what habits it involves. Now, the identity of a habit depends on how it might lead us to act, not merely under circumstances likely to arise, but under any that might possibly occur, no matter how improbable. What the habit is depends on *when* and *how* it causes us to act. As for the *when*, every stimulus to action is derived from perception; as for the *how*, every purpose of action is to produce some sensible result. Thus we come down to what is tangible and practical as the root of every real distinction of thought, no matter how subtle; and there is no distinction of meaning so fine as to consist in anything but a possible difference of practice.

2.15 To see where this principle leads, consider in its light a doctrine such as transubstantiation. The Protestant churches generally hold that the elements of the sacrament are flesh and blood only in a figurative sense: they nourish our souls as meat and its juice would nourish our bodies. But the Catholics maintain that they are literally just that, although they have all the sensible qualities of wafers and diluted wine. But we can have no conception of wine except what may enter into a belief, either—[8]

2.16 1. That this, that, or the other thing is wine; or, / 2. That wine possesses certain properties.

2.17 Such beliefs are nothing but notices to ourselves that, when the occasion arises, we should act with regard to things we believe to be wine according to the qualities we believe wine to have. The occasion of such action would be some sensible perception; its motive, to produce some sensible result. Thus our action refers exclusively to what affects the senses; our habit

8 Transubstantiation: the Catholic doctrine that in the Eucharist the bread and wine become in substance the body and blood of Christ, while keeping their outward appearance.

has the same bearing as our action, our belief the same as our habit, our conception the same as our belief. Consequently we can mean nothing by wine but what has certain effects, direct or indirect, on our senses; and to talk of something as having all the sensible characters of wine, yet being in reality blood, is senseless jargon. Now, it is not my aim to pursue the theological question; having used it as a logical example, I drop it, without caring to anticipate the theologian's reply. I only want to point out how impossible it is for us to have an idea in our minds that relates to anything but conceived sensible effects of things. Our idea of anything *is* our idea of its sensible effects; and if we imagine we have any other, we deceive ourselves and mistake a mere sensation accompanying the thought for a part of the thought itself. It is absurd to say that thought has any meaning unrelated to its only function. It is foolish for Catholics and Protestants to imagine themselves in disagreement about the elements of the sacrament, if they agree about all their sensible effects, here or hereafter.

2.18 It appears, then, that the rule for attaining the third grade of clearness of apprehension is as follows: Consider what effects, which might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object.

HARDNESS, WEIGHT AND FORCE

Peirce applies his rule to the diamond in cotton, free will, weight, and the idea of force.

2.19 Let us illustrate this rule with some examples. To begin with the simplest one possible, let us ask what we mean by calling a thing *hard*. Evidently, that it will not be scratched by many other substances. The whole conception of this quality, as of every other, lies in its conceived effects. There is absolutely no difference between a hard thing and a soft thing so long as they are not put to the test. Suppose, then, that a diamond could be crystallized in the middle of a cushion of soft cotton, and should

remain there until it was finally burned up. Would it be false to say that that diamond was soft? This seems a foolish question, and in fact it would be one, except in the realm of logic. There such questions are often extremely useful, because they bring logical principles into sharper relief than real discussions ever could. In studying logic we must not brush them aside with hasty answers, but must consider them with attentive care, in order to make out the principles involved. In the present case we may modify our question and ask what prevents us from saying that all hard bodies remain perfectly soft until they are touched, when their hardness increases with the pressure until they are scratched. Reflection will show that the answer is this: there would be no *falsity* in such ways of speaking. They would involve a change in our present usage of the words hard and soft, but not in their meanings. For they represent no fact as different from what it is; they only involve arrangements of the facts that would be exceedingly clumsy. This leads us to remark that the question of what would happen under circumstances that do not actually arise is not a question of fact, but only of the clearest arrangement of the facts. For example, the question of free will and fate in its simplest form, stripped of verbiage, is something like this: I have done something I am ashamed of; could I, by an effort of will, have resisted the temptation and done otherwise? The philosophical reply is that this is not a question of fact, but only of the arrangement of facts. If we arrange them so as to bring out what is particularly pertinent to my question—namely, that I ought to blame myself for having done wrong—it is perfectly true to say that, if I had willed to do otherwise than I did, I would have done otherwise. On the other hand, if we arrange the facts so as to bring out another important consideration, it is equally true that once a temptation has been allowed to work, it will, if it has a certain force, produce its effect, however I may struggle. There is no objection to a contradiction in what would result from a false supposition. The *reductio ad absurdum* consists in showing that contradictory results would follow from a hypothesis, which is consequently judged

to be false. Many questions are involved in the free-will discussion, and I am far from wishing to say that both sides are equally right. On the contrary, I am of the opinion that one side denies important facts and the other does not. But what I do say is that the single question above was the origin of the whole doubt; that, had it not been for this question, the controversy would never have arisen; and that this question is perfectly solved in the way I have indicated.[9]

2.20 Let us next seek a clear idea of Weight. This is another very easy case. To say that a body is heavy means simply that, in the absence of opposing force, it will fall. This (leaving aside certain specifications of how it will fall, etc., which exist in the mind of the physicist who uses the word) is evidently the whole conception of weight. It is a fair question whether some particular facts may not *account* for gravity; but what we mean by the force itself is completely involved in its effects.

2.21 This leads us to attempt an account of the idea of Force in general. This is the great conception which, developed in the early seventeenth century from the crude idea of a cause, and constantly improved since, has shown us how to explain all the changes of motion that bodies undergo, and how to think about all physical phenomena. It has given birth to modern science and changed the face of the globe; and, apart from its more special uses, it has played a principal part in directing the course of modern thought and in furthering modern social development. It is therefore worth some effort to understand it. According to our rule, we must begin by asking what the immediate use of thinking about force is; and the answer is that we thereby account for changes of motion. If bodies were left to themselves, without the intervention of forces, every motion would continue unchanged in both velocity and direction. Furthermore, change of motion never happens abruptly. If its direction changes, it is always through a curve without angles; if its velocity alters, it does so by degrees. Geometers conceive the gradual changes

9 *Reductio ad absurdum*: Latin, "reduction to absurdity," disproof of a supposition by deriving a contradiction from it.

constantly taking place as compounded together according to the rules of the parallelogram of forces. If the reader does not already know what this is, he will find it worth his while, I hope, to try to follow the explanation below. But if mathematics is unbearable to him, I beg him to skip three paragraphs rather than have us part company here.

2.22 A *path* is a line whose beginning and end are distinguished. Two paths are considered equivalent if, beginning at the same point, they lead to the same point. Thus the two paths *A B C D E* and *A F G H E* are equivalent. Paths that do not begin at the same point are considered equivalent provided that, if either of them is moved without turning it, keeping it always parallel to its original position, then when its beginning coincides with that of the other path, the ends also coincide. Paths are considered to be geometrically added together when one begins where the other ends. Thus the path *A E* is conceived to be the sum of *A B, B C, C D,* and *D E*. In the parallelogram of Fig. 4 the diagonal *A C* is the sum of *A B* and *B C;* or, since *A D* is geometrically equivalent to *B C, A C* is the geometrical sum of *A B* and *A D*.

2.23 All this is purely conventional. It simply amounts to this: we choose to call paths that have the relations I have described equal, or added. But though it is a convention, it is a convention with a good reason. The rule for geometrical addition may be applied not only to paths but to any other things that can be represented by paths. Now, a path is determined by the varying direction and distance from the starting point of the point that moves over it. It follows that anything which, from its beginning to its end, is determined by a varying direction and a varying magnitude can be represented by a line. Accordingly, *velocities* may be represented by lines, for they have only directions and rates. The same is true of *accelerations*, or changes of velocities. This is evident enough in the case of velocities. It becomes evident for accelerations if we consider that accelerations are to velocities precisely what velocities are to positions—namely, states of change of them.

2.24 The so-called "parallelogram of forces" is simply a rule for compounding accelerations. The rule is to represent the accelerations by paths and then to add the paths geometrically. The geometers, however, use the "parallelogram of forces" not only to compound different accelerations but also to resolve one acceleration into a sum of several. Let *A B* (Fig. 5) be the path that represents a certain acceleration—say, a change in the motion of a body such that, at the end of one second, the body will, under the influence of that change, be in a different position from the one it would have had if its motion had continued unchanged. It will differ in such a way that a path equivalent to *A B* would lead from the latter position to the former. This acceleration may be considered as the sum of the accelerations represented by *A C* and *C B*. It may also be considered as the sum of the very different accelerations represented by *A D* and *D B*, where *A D* is almost the opposite of *A C*.[10] And clearly there is an immense variety of ways in which *A B* might be resolved into the sum of two accelerations.

2.25 After this tedious explanation, which I hope has not exhausted the reader's patience, given the extraordinary interest of the conception of force, we are at last prepared to state the grand fact that this conception embodies. The fact is this. Suppose the actual changes of motion that the different particles of bodies undergo are each resolved in its appropriate way. Then each component acceleration is precisely such as a certain law of Nature prescribes. According to this law, bodies in the relative positions that the bodies in question actually have at the moment[11] always receive certain accelerations, and these, compounded by geometrical addition, give the acceleration that the body actually experiences.

2.26 This is the only fact that the idea of force represents, and whoever takes the trouble to grasp clearly what this fact is understands perfectly what force is. Whether we ought to say

10 The original prints "where A B is almost the opposite of A C," evidently a misprint for A D.

11 PEIRCE'S NOTE. Possibly the velocities also have to be taken into account.

that a force is an acceleration, or that it causes an acceleration, is a mere question of propriety of language. It has no more to do with our real meaning than the difference between the French idiom "*Il fait froid*" and its English equivalent "*It is cold*." Yet it is surprising to see how this simple affair has muddled men's minds. In how many profound treatises is force spoken of as a "mysterious entity"—which seems only a way of confessing that the author despairs of ever getting a clear notion of what the word means! In a recent, much-admired work on "Analytic Mechanics" it is stated that we understand precisely the effect of force, but what force itself is we do not understand! This is simply a self-contradiction. The idea that the word force excites in our minds has no other function than to affect our actions, and these actions can refer to force only through its effects. Consequently, if we know what the effects of force are, we are acquainted with every fact implied in saying that a force exists, and there is nothing more to know. The truth is that a vague notion is afloat that a question may mean something the mind cannot conceive. When some hairsplitting philosophers have been confronted with the absurdity of such a view, they have invented an empty distinction between positive and negative conceptions, in an attempt to give their non-idea a form that is not obviously nonsensical. Its nullity is plain enough from the considerations given a few pages back; and, apart from those considerations, the quibbling character of the distinction must have struck every mind accustomed to real thinking.

REALITY

The real is what is independent of what anyone thinks of it; the rule shows what this means.

2.27 Let us now approach the subject of logic and consider a conception that particularly concerns it, that of *reality*. Taking clearness in the sense of familiarity, no idea could be clearer than this. Every child uses it with perfect confidence, never dreaming that he does not understand it. As for clearness in its second

grade, however, most people, even those of a reflective turn of mind, would probably be puzzled to give an abstract definition of the real. Yet such a definition may perhaps be reached by considering the points of difference between reality and its opposite, fiction. A figment is a product of somebody's imagination; it has such characters as his thought impresses on it. That whose characters are independent of how you or I think is an external reality. There are, however, phenomena within our own minds, dependent on our thought, which are at the same time real in the sense that we really think them. But though their characters depend on how we think, they do not depend on what we think those characters to be. Thus a dream has a real existence as a mental phenomenon, if somebody has really dreamt it. That he dreamt such and such does not depend on what anybody thinks was dreamt, but is completely independent of all opinion on the subject. On the other hand, if we consider not the fact of dreaming but the thing dreamt, it retains its peculiarities by virtue of no other fact than that it was dreamt to possess them. Thus we may define the real as that whose characters are independent of what anybody may think them to be.

[2.28] But however satisfactory such a definition may be found, it would be a great mistake to suppose that it makes the idea of reality perfectly clear. Here, then, let us apply our rules. According to them, reality, like every other quality, consists in the peculiar sensible effects that things partaking of it produce. The only effect real things have is to cause belief, for all the sensations they excite emerge into consciousness in the form of beliefs. The question, therefore, is how true belief (or belief in the real) is distinguished from false belief (or belief in fiction). Now, as we saw in the previous paper, the ideas of truth and falsehood, in their full development, belong exclusively to the scientific method of settling opinion. A person who arbitrarily chooses the propositions he will adopt can use the word truth only to emphasize his determination to hold on to his choice. Of course, the method of tenacity never prevailed exclusively; reason is too natural to men for that. But in the literature of the

Dark Ages we find some fine examples of it. Scotus Erigena, commenting on a poetical passage in which hellebore is said to have caused the death of Socrates, does not hesitate to inform the inquiring reader that Helleborus and Socrates were two eminent Greek philosophers, and that the latter, having been overcome in argument by the former, took the matter to heart and died of it! What sort of idea of truth could a man have who could adopt and teach an opinion taken so entirely at random, without so much as a "perhaps"?[12] The real spirit of Socrates, who I hope would have been delighted to be "overcome in argument," because he would have learned something by it, contrasts curiously with the naive idea of the glossator, for whom discussion seems to have been simply a struggle. When philosophy began to awake from its long slumber, and before theology completely dominated it, the practice seems to have been for each professor to seize on any philosophical position he found unoccupied that seemed a strong one. He would entrench himself in it and sally forth from time to time to do battle with the others. Thus even the scanty records we possess of those disputes let us make out a dozen or more opinions held by different teachers at one time on the question of nominalism and realism. Read the opening part of the "Historia Calamitatum" of Abelard, who was certainly as philosophical as any of his contemporaries, and see the spirit of combat it breathes. For him, the truth is simply his particular stronghold. When the method of authority prevailed, the truth meant little more than the Catholic faith. All the efforts of the scholastic doctors are directed toward harmonizing their faith in Aristotle with their faith in the Church, and one may search their ponderous folios from cover to cover without finding an argument that goes any further.[13] It is noticeable that where different faiths flourish side by side, renegades are looked on with contempt even by the party whose belief

12 John Scotus Eriugena (c. 815–c. 877), Irish philosopher and theologian at the Carolingian court. Hellebore is a poisonous plant.

13 Peter Abelard (1079–1142); his "Historia Calamitatum" ("Story of My Misfortunes") is autobiographical. Nominalism and realism were the rival medieval positions on whether universals are real.

they adopt. So completely has the idea of loyalty replaced that of truth-seeking. Since the time of Descartes, the defect in the conception of truth has been less apparent. Still, it will sometimes strike a scientific man that the philosophers have been less intent on finding out what the facts are than on asking what belief is most in harmony with their system. It is hard to convince a follower of the *a priori* method by citing facts; but show him that an opinion he is defending is inconsistent with what he has laid down elsewhere, and he will be very apt to retract it. These minds do not seem to believe that disputation is ever to cease. They seem to think that the opinion natural for one man is not so for another, and that belief will consequently never be settled. In contenting themselves with fixing their own opinions by a method that would lead another man to a different result, they betray how feeble their hold is on the conception of what truth is.

TRUTH AS THE DESTINED OPINION

Investigation carries all inquirers to one fated opinion, and this is what truth and reality mean.

2.29 On the other hand, all the followers of science are fully persuaded that the processes of investigation, if only pushed far enough, will give one certain solution to every question to which they can be applied. One man may investigate the velocity of light by studying the transits of Venus and the aberration of the stars; another by the oppositions of Mars and the eclipses of Jupiter's satellites; a third by the method of Fizeau; a fourth by that of Foucault; a fifth by the motions of the curves of Lissajous. A sixth, a seventh, an eighth, and a ninth may follow the different methods of comparing the measures of static and dynamic electricity.[14] They may at first obtain different results,

14 Fizeau (1849) and Foucault (1862) measured the speed of light in the laboratory; Lissajous curves are figures traced by combined vibrations (the original spells the name "Lissajoux"). The ratio of electrostatic to electromagnetic units also yields the speed of light.

but as each perfects his method and his processes, the results will move steadily together toward a destined center. So with all scientific research. Different minds may set out with the most antagonistic views, but the progress of investigation carries them, by a force outside themselves, to one and the same conclusion. This activity of thought, by which we are carried not where we wish but to a foreordained goal, is like the operation of destiny. No change in the point of view taken, no selection of other facts for study, not even a natural bent of mind, can enable a man to escape the predestined opinion. This great law is embodied in the conception of truth and reality. The opinion which is fated[15] to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real. That is the way I would explain reality.

2.30 But it may be said that this view is directly opposed to the abstract definition of reality we have given, since it makes the characters of the real depend on what is ultimately thought about them. The answer is twofold. On the one hand, reality is independent, not necessarily of thought in general, but only of what you or I or any finite number of men may think about it. On the other hand, though the object of the final opinion depends on what that opinion is, what that opinion is does not depend on what you or I or any man thinks. Our perversity and that of others may postpone the settlement of opinion indefinitely; it might even conceivably cause an arbitrary proposition to be universally accepted as long as the human race should last. Yet even that would not change the nature of the belief, which alone could be the result of investigation carried sufficiently far. And if, after the extinction of our race, another should arise with the faculties and disposition for investigation, that true opinion must be the one they would ultimately come to. "Truth crushed to earth shall rise again," and the opinion that

15 PEIRCE'S NOTE. Fate means merely that which is sure to come true and can in no way be avoided. It is a superstition to suppose that a certain sort of events are ever fated, and it is another to suppose that the word fate can never be freed from its superstitious taint. We are all fated to die.

would finally result from investigation does not depend on how anybody may actually think.[16] But the reality of that which is real does depend on the real fact that investigation is destined to lead at last, if continued long enough, to a belief in it.

2.31 But I may be asked what I have to say about all the minute facts of history, forgotten never to be recovered, about the lost books of the ancients, about the buried secrets.

2.32 "Full many a gem of purest ray serene
The dark, unfathomed caves of ocean bear;
Full many a flower is born to blush unseen,
And waste its sweetness on the desert air."[17]

2.33 Do these things not really exist because they are hopelessly beyond the reach of our knowledge? And then, after the universe is dead (as some scientists predict) and all life has ceased forever, will not the shock of atoms continue though there is no mind to know it? To this I reply as follows. In no possible state of knowledge can any number be great enough to express the ratio of the amount of what remains unknown to the amount of the known. Yet it is unphilosophical to suppose that, with regard to any given question (that has any clear meaning), investigation would not bring forth a solution to it, if it were carried far enough. Who would have said, a few years ago, that we could ever know what substances make up stars whose light may have taken longer to reach us than the human race has existed? Who can be sure what we shall not know in a few hundred years? Who can guess what would come of continuing the pursuit of science for ten thousand years with the activity of the last hundred? And if it were to go on for a million, or a billion, or any number of years you please, how is it possible to say that there is any question that might not ultimately be solved?[18]

2.34 But it may be objected: "Why make so much of these remote considerations, especially when it is your principle that

16 The quoted line is from William Cullen Bryant's poem "The Battle-Field" (1839).
17 From Thomas Gray, "Elegy Written in a Country Churchyard" (1751).
18 The chemical composition of stars was first determined by spectral analysis, developed by Kirchhoff and Bunsen around 1859.

only practical distinctions have a meaning?" Well, I must confess that it makes very little difference whether we say that a stone at the bottom of the ocean, in complete darkness, is brilliant or not—that is to say, that it *probably* makes no difference, always remembering that the stone *may* be fished up tomorrow. But that there are gems at the bottom of the sea, flowers in the untraveled desert, and so on, are propositions which, like the one about a diamond being hard when it is not pressed, concern the arrangement of our language much more than the meaning of our ideas.

2.35 It seems to me, however, that by applying our rule we have reached so clear an apprehension of what we mean by reality, and of the fact on which the idea rests, that we should not perhaps be making a claim as presumptuous as it would be singular if we were to offer a metaphysical theory of existence for universal acceptance among those who employ the scientific method of fixing belief. However, metaphysics is a subject much more curious than useful, and knowledge of it, like knowledge of a sunken reef, serves chiefly to enable us to keep clear of it. So I will not trouble the reader with any more Ontology at this moment. I have already been led much further along that path than I should have wished. I have given the reader such a dose of mathematics, psychology, and all that is most abstruse, that I fear he may already have left me, and that what I am now writing is for the typesetter and proofreader alone. I trusted to the importance of the subject. There is no royal road to logic, and really valuable ideas can be had only at the price of close attention. But I know that in the matter of ideas the public prefers the cheap and nasty; and in my next paper I am going to return to the easily intelligible and not wander from it again. The reader who has taken the pains to wade through this month's paper shall be rewarded in the next one by seeing how beautifully what has been developed in this tedious way can be applied to ascertaining the rules of scientific reasoning.

2.36 So far we have not crossed the threshold of scientific logic. It is certainly important to know how to make our ideas clear,

but they may be ever so clear without being true. How to make them true we have next to study. How to give birth to those vital and procreative ideas which multiply into a thousand forms and spread everywhere, advancing civilization and making the dignity of man, is an art not yet reduced to rules; but the history of science affords some hints of its secret.

The Doctrine of Chances

Probability as the ratio of frequencies in the long run, and the logic of an unlimited community of inquirers.

THE DOCTRINE OF CHANCES I.

3.1 It is a common observation that a science first begins to be exact when it is treated quantitatively. What are called the exact sciences are simply the mathematical ones. Chemists reasoned vaguely until Lavoisier showed them how to apply the balance to verifying their theories; then chemistry leaped suddenly into the position of the most perfect of the classificatory sciences. It has thus become so precise and certain that we usually think of it along with optics, the science of heat, and electricity. But these are studies of general laws, while chemistry considers merely the relations and classification of certain objects. In reality it belongs in the same category as systematic botany and zoology. Compare it with these last, however, and the advantage it derives from its quantitative treatment is very evident.

3.2 The crudest numerical scales, such as the one by which mineralogists distinguish the different degrees of hardness, prove useful. The mere counting of pistils and stamens sufficed to bring botany out of total chaos into some kind of form. It is not so much from *counting*, however, as from *measuring*, not so much from the conception of number as from that of continuous quantity, that the advantage of mathematical treatment comes. Number, after all, only serves to pin us down to a precision in our thoughts which, however beneficial, can seldom lead

to lofty conceptions and frequently descends to pettiness.[1] Bacon speaks of two faculties, the one that marks differences and the one that notes resemblances; the use of number can aid only the lesser of the two, and the excessive use of it must tend to narrow the powers of the mind. But the conception of continuous quantity has a great office to fulfill, quite apart from any attempt at precision. Far from tending to exaggerate differences, it is the direct instrument of the finest generalizations. When a naturalist wishes to study a species, he collects a considerable number of more or less similar specimens. In contemplating them, he notices certain ones that are more or less alike in some particular respect. They all have, for instance, a certain S-shaped marking. He observes that they are not *precisely* alike in this respect; the S has not precisely the same shape. But the differences are such as to lead him to believe that forms could be found intermediate between any two of those he possesses. He now finds other forms apparently quite dissimilar—say, a marking in the form of a C—and the question is whether he can find intermediate ones that will connect these with the others. He often succeeds in doing this in cases where it would at first be thought impossible; whereas he sometimes finds that forms which at first glance differ much less are separated in Nature by the absence of intermediates. In this way he builds up, from the study of Nature, a new general conception of the character in question. He obtains, for example, an idea of a leaf that includes every part of the flower, and an idea of a vertebra that includes the skull. I surely need not say much to show what a logical engine there is here. It is the essence of the naturalist's method. How he applies it first to one character and then to another, and finally obtains a notion of a species of animals the differences between whose members, however great, are confined within limits, is a matter that does not concern us here.[2] The whole

1 The hardness scale is Mohs's (1812); the classification of plants by counting stamens and pistils is Linnaeus's.

2 The leaf theory of the flower and the vertebral theory of the skull were proposed by Goethe and by Lorenz Oken.

method of classification must be considered later. At present I only wish to point out that it is by taking advantage of the idea of continuity, or the passage from one form to another by insensible degrees, that the naturalist builds his conceptions. Now, the naturalists are the great builders of conceptions; in no other branch of science is so much of this work done as in theirs; and we must, in great measure, take them as our teachers in this important part of logic. And it will be found everywhere that the idea of continuity is a powerful aid to forming true and fruitful conceptions. By means of it, the greatest differences are broken down and resolved into differences of degree, and its incessant application is of the greatest value in broadening our conceptions. I propose to make great use of this idea in the present series of papers. The particular series of important fallacies that have arisen from neglecting it, and have desolated philosophy, must be closely studied further on. At present I simply call the reader's attention to the usefulness of this conception.

3.3 In studies of numbers, the idea of continuity is so indispensable that it is perpetually introduced even where there is no continuity in fact—as when we say that there are 10.7 inhabitants per square mile in the United States, or that in New York 14.72 persons live in the average house.[3] Another example is the law of the distribution of errors which Quetelet, Galton, and others have applied so successfully to the study of biological and social matters. This application of continuity to cases where it does not really exist also illustrates another point that will later demand a separate study, namely, the great usefulness that fictions sometimes have in science.[4]

3 PEIRCE'S NOTE. This mode of thought is so familiarly associated with all exact numerical consideration that shallow writers imitate the phrase appropriate to it in order to produce the appearance of exactitude where none exists. Certain newspapers that affect a learned tone talk of "the average man" when they simply mean *most men*, and have no idea of striking an average.

4 Adolphe Quetelet (1796–1874), Belgian statistician, and Francis Galton (1822–1911), English scientist, applied the normal law of errors to human traits.

PROBABILITY AS QUANTITATIVE LOGIC

The theory of probabilities is logic treated quantitatively, though its first principles are still disputed.

3.4 The theory of probabilities is simply the science of logic treated quantitatively. There are two conceivable certainties with reference to any hypothesis: the certainty of its truth and the certainty of its falsity. In this calculus, the numbers *one* and *zero* are assigned to mark these extremes of knowledge, while fractions with values between them indicate, as we may vaguely say, the degrees to which the evidence leans toward one or the other. The general problem of probabilities is, from a given state of facts, to determine the numerical probability of a possible fact. This is the same as asking how much the given facts are worth, considered as evidence to prove the possible fact. Thus the problem of probabilities is simply the general problem of logic.

3.5 Probability is a continuous quantity, so great advantages may be expected from this way of studying logic. Some writers have gone so far as to maintain that, by means of the calculus of chances, every solid inference may be represented by legitimate arithmetical operations on the numbers given in the premises. If this is indeed true, the great problem of logic—how the observation of one fact can give us knowledge of another, independent fact—is reduced to a mere question of arithmetic. It seems proper to examine this claim before undertaking any more recondite solution of the paradox.

3.6 But unfortunately, writers on probabilities are not agreed about this result. This branch of mathematics is the only one, I believe, in which good writers frequently get results that are entirely wrong. In elementary geometry the reasoning is frequently fallacious, but erroneous conclusions are avoided. In probabilities, however, it may be doubted whether there exists a single extensive treatise that does not contain solutions absolutely indefensible. This is partly owing to the lack of any regu-

lar method of procedure, for the subject involves too many subtleties to make it easy to put its problems into equations without such an aid. But beyond this, the fundamental principles of its calculus are more or less in dispute. About the class of questions to which it is chiefly applied for practical purposes, there is comparatively little doubt; but about others to which people have sought to extend it, opinion is somewhat unsettled.

3.7 This last class of difficulties can be entirely overcome only by making the idea of probability perfectly clear in our minds, in the way set forth in our last paper.

WHAT PROBABILITY MEANS

Probability is the proportion of cases in which a kind of inference carries truth, in the long run.

3.8 To get a clear idea of what we mean by probability, we have to consider what real and sensible difference there is between one degree of probability and another.

3.9 The character of probability belongs primarily, without doubt, to certain inferences. Locke explains it as follows. After remarking that the mathematician positively knows that the sum of the three angles of a triangle is equal to two right angles because he apprehends the geometrical proof, he continues: "But another man who never took the pains to observe the demonstration, hearing a mathematician, a man of credit, affirm the three angles of a triangle to be equal to two right ones, *assents* to it; i.e., receives it for true. In which case the foundation of his assent is the probability of the thing, the proof being such as, for the most part, carries truth with it; the man on whose testimony he receives it not being wont to affirm anything contrary to, or besides his knowledge, especially in matters of this kind." The celebrated "Essay concerning Humane Understanding" contains many passages which, like this one, take the first steps in profound analyses that are not developed further.[5] It

5 John Locke (1632–1704), *An Essay Concerning Human Understanding* (1690), book IV, chapter 15, section 1.

was shown in the first of these papers that the validity of an inference does not depend on any tendency of the mind to accept it, however strong that tendency may be. It consists in the real fact that, when premises like those of the argument in question are true, conclusions related to them as this argument's conclusion is are also true. It was remarked that in a logical mind an argument is always conceived as a member of a *genus* of arguments, all constructed in the same way, such that when their premises are real facts, their conclusions are so too. If the argument is demonstrative, this is always so; if it is only probable, it is so for the most part. As Locke says, the probable argument is "*such as* for the most part carries truth with it."

3.10 According to this, the real and sensible difference between one degree of probability and another, in which the meaning of the distinction lies, is that, when two different modes of inference are frequently employed, one will carry truth with it more often than the other. Evidently this is the only difference there is in the existing fact. Having certain premises, a man draws a certain conclusion. As far as this inference alone is concerned, the only possible practical question is whether that conclusion is true or not, and between existence and non-existence there is no middle term. "Being only is and nothing is altogether not," said Parmenides; and this is in strict accordance with the analysis of the conception of reality given in the last paper. For we found that the distinction between reality and fiction depends on the supposition that sufficient investigation would cause one opinion to be universally received and all others to be rejected. That presupposition, involved in the very conceptions of reality and figment, implies a complete sundering of the two. It is the heaven-and-hell idea in the domain of thought.[6] But in the long run there is a real fact that corresponds to the idea of probability: a given mode of inference sometimes proves successful and sometimes not, and that in a ratio ultimately fixed. As we go on drawing inference after inference of the given kind, during the

6 Parmenides of Elea (early 5th century BC), Greek philosopher who held that what is, is, and what is not cannot be.

first ten or hundred cases the ratio of successes may be expected to show considerable fluctuations. But when we come into the thousands and millions, these fluctuations become less and less; and if we continue long enough, the ratio will approach a fixed limit. We may therefore define the probability of a mode of argument as the proportion of cases in which it carries truth with it.

3.11 The inference from the premise A to the conclusion B depends, as we have seen, on the guiding principle that if a fact of the class A is true, a fact of the class B is true. The probability consists of the fraction whose numerator is the number of times in which both A and B are true, and whose denominator is the total number of times in which A is true, whether B is or not. Instead of speaking of this as the probability of the inference, there is not the slightest objection to calling it the probability that, if A happens, B happens. But to speak of the probability of the event B, without naming the condition, really has no meaning at all. It is true that when it is perfectly obvious what condition is meant, the ellipsis may be permitted. But we should avoid falling into the habit of using language this way (universal as the habit is). It gives rise to a vague way of thinking, as if the action of causation might either determine an event to happen, or determine it not to happen, or leave it more or less free to happen or not, so as to give rise to an *inherent* chance regarding its occurrence. It is quite clear to me that some of the worst and most persistent errors in the use of the doctrine of chances have arisen from this vicious mode of expression.[7]

7 PEIRCE'S NOTE. The conception of probability set forth here is substantially the one first developed by Mr. Venn in his "Logic of Chance." Of course, a vague apprehension of the idea had always existed, but the problem was to make it perfectly clear, and to him belongs the credit of first doing so.

TRANSLATOR'S NOTE. John Venn (1834–1923), English logician; *The Logic of Chance* appeared in 1866.

THE SINGLE CASE

Probability means nothing in a single case, which seems to leave a once-only choice without reasons.

3.12 But an important point remains to be cleared up. According to what has been said, the idea of probability belongs essentially to a kind of inference that is repeated indefinitely. An individual inference must be either true or false, and can show no effect of probability. Therefore, in reference to a single case considered in itself, probability can have no meaning. Yet suppose a man had to choose between drawing a card from a pack containing twenty-five red cards and one black one, or from a pack containing twenty-five black cards and one red one. Suppose, too, that drawing a red card were destined to transport him to eternal felicity, and drawing a black one to consign him to everlasting woe. It would be folly to deny that he ought to prefer the pack containing the larger proportion of red cards, although, from the nature of the risk, it could not be repeated. It is not easy to reconcile this with our analysis of the conception of chance. But suppose he chose the red pack and drew the wrong card: what consolation would he have? He might say that he had acted in accordance with reason, but that would only show that his reason was absolutely worthless. And if he drew the right card, how could he regard it as anything but a happy accident? He could not say that if he had drawn from the other pack he might have drawn the wrong one, because an hypothetical proposition such as "if A, then B" means nothing with reference to a single case. Truth consists in the existence of a real fact corresponding to the true proposition. Corresponding to the proposition "if A, then B," there may be the fact that *whenever* such an event as A happens, such an event as B happens. But in the case supposed, which has no parallel as far as this man is concerned, there would be no real fact whose existence could give any truth to the statement that, if he had drawn from the other pack, he might have drawn a black card. Indeed, the validity of an infer-

ence consists in the truth of the hypothetical proposition that *if* the premises are true, the conclusion will also be true. And the only real fact that can correspond to such a proposition is that whenever the antecedent is true, the consequent is so too. It follows that there can be no sense at all in reasoning in an isolated case.

3.13 At first sight, these considerations appear to dispose of the difficulty mentioned. Yet the other side has not yet made its whole case. Probability will probably show its effect in, say, a thousand risks, through a certain proportion between the numbers of successes and failures. But this, as we have seen, only amounts to saying that it certainly will do so in the end. Now the number of risks, the number of probable inferences, that a man draws in his whole life is finite, and he cannot be absolutely *certain* that the mean result will accord with the probabilities at all. Taking all his risks together, then, it cannot be certain that they will not fail, and his case differs only in degree from the one last supposed. It is an indubitable result of the theory of probabilities that every gambler, if he keeps playing long enough, must ultimately be ruined. Suppose he tries the martingale, which some believe infallible and which, I am told, is not allowed in the gambling-houses. In this method of play, he first bets, say, $1. If he loses it, he bets $2; if he loses that, he bets $4; if he loses that, he bets $8. If he then wins, he has lost 1 + 2 + 4 = 7, and he has gained $1 more. No matter how many bets he loses, the first one he wins will leave him $1 richer than he was at the beginning. In this way he will probably gain at first. But at last the time will come when the run of luck is so much against him that he will not have enough money to double, and must therefore let his bet go. This will *probably* happen before he has won as much as he had to begin with, so that this run against him will leave him poorer than when he started. At some time or other it is sure to happen. It is true that there is always a possibility of his winning any sum the bank can pay, and so we come upon a celebrated paradox: though he is certain to be ruined, the value of his expectation, calculated by the usual rules (which leave

this consideration out), is large.[8] But whether a gambler plays this way or any other, the same thing is true: if he plays long enough, he is sure at some point to meet such a run against him as to exhaust his entire fortune. The same is true of an insurance company. Let the directors take the utmost pains to be independent of great fires and epidemics. Their actuaries can still tell them that, according to the doctrine of chances, the time must come at last when their losses will bring them to a stop. They may tide over such a crisis by extraordinary means, but then they will start again in a weakened state, and the same thing will happen again all the sooner. An actuary might be inclined to deny this, because he knows that his company's expectation is large, or perhaps (neglecting interest on money) infinite. But calculations of expectations leave out of account the circumstance we are now considering, and that circumstance reverses the whole thing. However, I must not be understood as saying that insurance is on this account any more unsound than other kinds of business. All human affairs rest on probabilities, and the same thing is true everywhere. If man were immortal, he could be perfectly sure of seeing the day when everything in which he had trusted would betray his trust. In short, he could be sure of coming eventually to hopeless misery. He would break down at last, as every great fortune, every dynasty, every civilization does. In place of this we have death.

LOGIC AND THE UNLIMITED COMMUNITY

Because death makes our inferences finite, logic requires interests that reach an unlimited community.

3.14 But what would happen to every man without death must, with death, happen to some man. At the same time, death makes the number of our risks, of our inferences, finite, and so makes their mean result uncertain. The very idea of probability and of reasoning rests on the assumption that this number is indefin-

8 "Expectation" is the mathematical expectation: the sum of the possible gains, each weighted by its probability.

itely great. We are thus landed in the same difficulty as before, and I can see only one solution of it. It seems to me that we are driven to this, that logicality inexorably requires that our interests shall *not* be limited. They must not stop at our own fate, but must embrace the whole community. This community, in turn, must not be limited, but must extend to all races of beings with whom we can come into direct or indirect intellectual relation. It must reach, however vaguely, beyond this geological epoch, beyond all bounds. He who would not sacrifice his own soul to save the whole world is, it seems to me, illogical in all his inferences, taken collectively. Logic is rooted in the social principle.

3.15 To be logical, men should not be selfish; and in point of fact they are not as selfish as they are thought to be. The willful pursuit of one's desires is a different thing from selfishness. The miser is not selfish: his money does him no good, and he cares about what will become of it after his death. We constantly speak of *our* possessions on the Pacific and of *our* destiny as a republic, where no personal interests are involved, in a way that shows we have wider interests. We discuss anxiously the possible exhaustion of coal in some hundreds of years, or the cooling of the sun in some millions. And in the most popular of all religious tenets we show that we can conceive the possibility of a man's descending into hell for the salvation of his fellows.[9]

3.16 Now, it is not necessary for logicality that a man should himself be capable of the heroism of self-sacrifice. It is enough that he should recognize its possibility, should see that only the inferences of the man who has it are really logical, and should therefore regard his own inferences as valid only so far as they would be accepted by the hero. So far as he thus refers his inferences to that standard, he becomes identified with such a mind.

3.17 This makes logicality attainable enough. Sometimes we can personally attain heroism. The soldier who runs to scale a wall knows that he will probably be shot, but that is not all he

9 An allusion to the Christian doctrine of Christ's descent into hell after the crucifixion.

cares about. He also knows that if the whole regiment, with which he identifies himself in feeling, rushes forward at once, the fort will be taken. In other cases we can only imitate the virtue. Take the man we imagined as having to draw from the two packs. If he is no logician, he will draw from the red pack out of mere habit. If he is logician enough, he will see that he cannot be logical so long as he is concerned only with his own fate. He will also see that a man who cared equally about what was to happen in all possible cases of the sort could act logically, and would draw from the pack with the most red cards. And so, though himself incapable of such sublimity, our logician would imitate the effect of that man's courage in order to share his logicality.

3.18 But all this requires that one conceive one's interests as identified with those of an unlimited community. Now, there are no reasons, and a later discussion will show that there can be no reasons, for thinking that the human race, or any intellectual race, will exist forever. On the other hand, there can be no reason against it;[10] and fortunately, since the whole requirement is that we should have certain sentiments, there is nothing in the facts to forbid our having a *hope*, or calm and cheerful wish, that the community may last beyond any assignable date.

3.19 It may seem strange that I should put forward three sentiments as indispensable requirements of logic: interest in an indefinite community, recognition of the possibility of this interest being made supreme, and hope in the unlimited continuance of intellectual activity. Yet logic depends on a mere struggle to escape doubt, and since that struggle ends in action, it must begin in emotion. Furthermore, the only cause of our taking our stand on reason is that other methods of escaping doubt fail because of the social impulse. When we consider this, why should we be surprised to find social sentiment presup-

10 PEIRCE'S NOTE. I do not here admit an absolutely unknowable. Evidence could show us what would probably be the case after any given lapse of time. And though a later time might be assigned which that evidence might not cover, further evidence would cover it.

posed in reasoning? As for the other two sentiments that I find necessary, they are so only as supports and accessories of that one. It interests me to notice that these three sentiments seem to be much the same as that famous trio of Charity, Faith, and Hope, which St. Paul regarded as the finest and greatest of spiritual gifts. Neither the Old nor the New Testament is a textbook of the logic of science, but the New is certainly the highest existing authority on the dispositions of heart that a man ought to have.[11]

RULES FOR CALCULATING CHANCES

Probability is a relative number, and rules for adding and multiplying such numbers apply to it.

3.20 Consider such average statistical numbers as the number of inhabitants per square mile, the average number of deaths per week, the number of convictions per indictment, or, in general, the number of *x*'s per *y*, where the *x*'s are a class of things some or all of which are connected with another class of things, their *y*'s. Numbers like these I call *relative numbers*. Of the two classes of things to which a relative number refers, the one of which it is a number may be called its *relate*, and the one per which the counting is made may be called its *correlate*.

3.21 Probability is a kind of relative number. It is the ratio of the number of arguments of a certain genus that carry truth with them to the total number of arguments of that genus, and the rules for calculating probabilities are very easily derived from this. They may all be given here, since they are extremely simple, and it is sometimes convenient to know something of the elementary rules for calculating chances.

3.22 Rule I. *Direct Calculation.*—To calculate any relative number directly (say, for instance, the number of passengers on the average trip of a streetcar), we must proceed as follows:

3.23 Count the number of passengers on each trip, add all these numbers, and divide by the number of trips. There are cases in

11 See 1 Corinthians 13, where "charity" translates the Greek word for love.

which this rule can be simplified. Suppose we wish to know the number of inhabitants per dwelling in New York. The same person cannot inhabit two dwellings; if he divides his time between two dwellings, he ought to be counted as half an inhabitant of each. In this case we have only to divide the total number of inhabitants of New York by the number of their dwellings, with no need to count separately those who inhabit each one. A similar procedure will apply wherever each individual relate belongs exclusively to one individual correlate. If we want the number of *x*'s per *y*, and no *x* belongs to more than one *y*, we have only to divide the whole number of *x*'s of *y*'s by the number of *y*'s. Such a method would, of course, fail if applied to finding the average number of streetcar passengers per trip. We could not divide the total number of travelers by the number of trips, since many of them would have made many journeys. To find the probability that a given class of conclusions, B, follows from a given class of premises, A, we simply need to find out in what proportion of the times when premises of that class are true the appropriate conclusions are also true. In other words, it is the number of cases in which both the events A and B occur, divided by the total number of cases in which the event A occurs.

3.24 Rule II.—*Addition of Relative Numbers.*—Suppose we are given two relative numbers having the same correlate, say the number of *x*'s per *y* and the number of *z*'s per *y*, and we are required to find the number of *x*'s and *z*'s together per *y*. If there is nothing that is at once an *x* and a *z* to the same *y*, the sum of the two given numbers gives the required number. Suppose, for example, that we were given the average number of friends men have and the average number of their enemies. The sum of these two is the average number of persons interested in a man. On the other hand, it plainly would not do to add together the average number of persons having constitutional diseases, the average number over military age, and the average number exempted from military service by each special cause, in order to get the average number exempt in any way, since many are exempt in two or more ways at once.

3.25 This rule applies directly to probabilities. Suppose we are given the probabilities that two different and mutually exclusive events will happen under the same supposed set of circumstances. Suppose, for instance, we are given the probability that if A then B, and also the probability that if A then C. Then the sum of these two probabilities is the probability that if A then either B or C, so long as there is no event that belongs at once to both classes B and C.

3.26 Rule III.—*Multiplication of Relative Numbers.*—Suppose we are given the relative number of x's per y, and also the relative number of z's per x of y. To take a concrete example, suppose we are given, first, the average number of children in families living in New York, and second, the average number of teeth in the head of a New York child. Then the product of these two numbers gives the average number of children's teeth in a New York family. But in general this way of reckoning applies only under two restrictions. In the first place, it would not be true if the same child could belong to different families. In that case the children who belonged to several different families might have an exceptionally large or small number of teeth, which would affect the average number of children's teeth in a family more than it would affect the average number of teeth in a child's head. In the second place, the rule would not be true if different children could share the same teeth, since the average number of children's teeth would then evidently be something different from the average number of teeth belonging to a child.

3.27 To apply this rule to probabilities, we must proceed as follows. Suppose we are given the probability that the conclusion B follows from the premise A, B and A representing, as usual, certain classes of propositions. Suppose we also knew the probability of an inference in which B would be the premise and a proposition of a third kind, C, the conclusion. Here, then, we have the materials for applying this rule. We have, first, the relative number of B's per A. We next should have the relative number of C's per B following from A. But suppose the classes of propositions are so selected that the probability of C's following from

any B in general is just the same as the probability of C's following from one of those B's that is deducible from an A. Then the two probabilities may be multiplied together to give the probability of C's following from A. The same restrictions exist as before. It might happen that the probability that B follows from A was affected by certain propositions of class B following from several different propositions of class A. But practically speaking, all these restrictions are of very little consequence. It is usually recognized as a universally true principle that the probability that, if A is true, B is, multiplied by the probability that, if B is true, C is, gives the probability that, if A is true, C is.

3.28 There is a rule supplementary to this, of which great use is made. It is not universally valid, and the greatest caution must be exercised in using it. The care needed is double: first, never to use it when it will involve serious error; and second, never to fail to take advantage of it in cases where it can be employed. This rule depends on the fact that in very many cases the probability that C is true if B is, is substantially the same as the probability that C is true if A is. Suppose, for example, we have the average number of males among the children born in New York; suppose that we also have the average number of children born in the winter months among those born in New York. Now we may assume without doubt, at least as a close approximation (and no very fine calculation would be in place where probabilities are concerned), that the proportion of males among all the children born in New York is the same as the proportion of males among those born in New York in summer. Therefore, if the names of all the children born during a year were put into an urn, we might multiply the probability that any name drawn would be that of a male child by the probability that it would be that of a child born in summer. This would give us the probability that it would be the name of a male child born in summer.[12] The questions of probability in the treatises on the subject have usually concerned balls drawn from urns, games of cards, and

12 Peirce shifts here from "winter" in the previous sentence to "summer"; the point is the same either way.

the like. In these the question of the *independence* of events, as it is called, has been very simple: that is, the question whether the probability of C under the hypothesis B is the same as its probability under the hypothesis A. But in applying probabilities to the ordinary questions of life, it is often an exceedingly delicate question whether two events may be considered independent with sufficient accuracy or not. In all calculations about cards it is assumed that the cards are thoroughly shuffled, which makes one deal quite independent of another. In fact, in practice the cards are seldom shuffled well enough for this to be true. In a game of whist, for example, the cards fall in fours of the same suit and are gathered up that way, so they will lie more or less in sets of four of the same suit, and this will be true even after they are shuffled.[13] At least some traces of this arrangement will remain. As a result, the number of "short suits," as they are called (that is, the number of hands in which the cards are very unequally divided among the suits), is smaller than calculation would make it. So when there is a misdeal, and the cards, thrown about the table, get very thoroughly shuffled, it is a common saying that the hands dealt next generally have short suits. A few years ago a friend of mine who plays a great deal of whist was kind enough to count the number of spades dealt to him in 165 hands, in which the cards had been, if anything, shuffled better than usual. According to calculation, there should have been 85 of these hands in which my friend held either three or four spades. In fact there were 94, which shows the influence of imperfect shuffling.

3.29 On the view taken here, these are the only fundamental rules for the calculation of chances. An additional one, derived from a different conception of probability, is given in some treatises; if it were sound, it might be made the basis of a theory of reasoning. I believe it is absolutely absurd, and for that reason considering it serves to bring us to the true theory. It is for the sake of this discussion, which must be postponed to the next issue, that I have brought the doctrine of chances to the

13 Whist: a four-player trick-taking card game, the ancestor of bridge.

reader's attention at this early stage of our studies of the logic of science.[14]

14 The rule meant is the rule of "inverse probability" (Bayes, Laplace), criticized in the next paper.

The Probability of Induction

Against the conceptualist theory of probability: why we cannot assign probabilities to hypotheses, and what induction can promise.

THE PROBABILITY OF INDUCTION I.

4.1 We have found that every argument derives its force from the general truth of the class of inferences to which it belongs, and that probability is the proportion of arguments carrying truth with them among those of any *genus.* This is most conveniently expressed in the terminology of the medieval logicians. They called the fact expressed by a premise an *antecedent,* and what follows from it its consequent. The leading principle, that every (or almost every) such antecedent is followed by such a *consequent,* they called the *consequence.* In this language, we may say that probability belongs exclusively to *consequences,* and that the probability of any consequence is the number of times in which antecedent and consequent both occur, divided by the number of all the times in which the antecedent occurs. From this definition the following rules for the addition and multiplication of probabilities are deduced:[1]

4.2 *Rule for the Addition of Probabilities.*—Suppose we are given the separate probabilities of two consequences having the same antecedent and incompatible consequents. Then the sum of these two numbers is the probability of the consequence that,

1 "Consequence" (Latin *consequentia*): in scholastic logic, the relation or rule linking an antecedent to its consequent.

from the same antecedent, one or the other of those consequents follows.

4.3 *Rule for the Multiplication of Probabilities.*—Suppose we are given the separate probabilities of the two consequences "If A then B" and "If both A and B, then C." Then the product of these two numbers is the probability of the consequence "If A, then both B and C."

4.4 *Special Rule for the Multiplication of Independent Probabilities.* —Suppose we are given the separate probabilities of two consequences having the same antecedents, "If A, then B" and "If A, then C." Suppose that these consequences are such that the probability of the second is equal to the probability of the consequence "If both A and B, then C." Then the product of the two given numbers is equal to the probability of the consequence "If A, then both B and C."

4.5 To show how these rules work, we may examine the probabilities in throwing dice. What is the probability of throwing a six with one die? The antecedent here is the event of throwing a die; the consequent is its turning up a six. Since the die has six sides, all of which turn up equally often, the probability of turning up any one is 1/6. Suppose two dice are thrown: what is the probability of throwing sixes? The probability of either coming up six is obviously the same when both are thrown as when one is thrown—namely, 1/6. The probability that either will come up six when the other does is also the same as the probability of its coming up six whether the other does or not. The probabilities are therefore independent, and by our rule the probability that both events will happen together is the product of their separate probabilities, 1/6 × 1/6. What is the probability of throwing deuce-ace? The probability that the first die will turn up ace and the second deuce is the same as the probability that both will turn up sixes—namely, 1/36. The probability that the *second* will turn up ace and the *first* deuce is likewise 1/36. These two events (first, ace, second, deuce; and second, ace, first, deuce) are incompatible.[2] Hence the rule for addition holds, and the prob-

2 Deuce-ace: a throw of a two and a one.

ability that one will come up ace and the other deuce is 1/36 + 1/36, or 1/18.

4.6 In this way all problems about dice and the like may be solved. When the number of dice thrown is supposed to be very large, mathematics (which may be defined as the art of making groups to facilitate counting) comes to our aid with certain devices for reducing the difficulties.

THE CONCEPTUALISTIC VIEW

Probability as degree of belief, measured by the logarithm of the chance, and why that view falls short.

4.7 The conception of probability as a matter of *fact* (that is, as the proportion of times in which an occurrence of one kind is accompanied by an occurrence of another kind) is called by Mr. Venn the materialistic view of the subject. But probability has often been regarded as simply the degree of belief that ought to attach to a proposition, and Venn calls this way of explaining the idea the conceptualistic view. Most writers have mixed the two conceptions together. They first define the probability of an event as the reason we have to believe that it has taken place, which is conceptualistic. But shortly afterward they state that it is the ratio of the number of cases favorable to the event to the total number of cases favorable or contrary, all being equally possible. This brings in the thoroughly unclear idea of cases equally possible in place of cases equally frequent, but apart from that it is a tolerable statement of the materialistic view. The pure conceptualistic theory has been best expounded by Mr. De Morgan in his "Formal Logic: or, the Calculus of Inference, Necessary and Probable."[3]

4.8 The great difference between the two analyses is that the conceptualists refer probability to an event, while the materialists make it the ratio of the frequency of events of a *species* to

3 Augustus De Morgan (1806–1871), English mathematician and logician; his *Formal Logic* appeared in 1847.

those of a *genus* over that species, thus *giving it two terms instead of one*. The opposition may be brought out as follows:

4.9 Suppose we have two rules of inference such that, of all the questions to whose solution both can be applied, the first yields correct answers to 81/100 and incorrect answers to the remaining 19/100, while the second yields correct answers to 93/100 and incorrect answers to the remaining 7/100. Suppose, further, that the two rules are entirely independent as to their truth. That is, the second answers correctly 93/100 of the questions that the first answers correctly, and also 93/100 of the questions that the first answers incorrectly. And it answers incorrectly the remaining 7/100 of the questions that the first answers correctly, and also the remaining 7/100 of the questions that the first answers incorrectly. Then, of all the questions to whose solution both rules can be applied—

4.10 both answer correctly 93/100 of 81/100 or (93 × 81)/(100 × 100);
the second answers correctly and the first incorrectly 93/100 of 19/100 or (93 × 19)/(100 × 100);
the second answers incorrectly and the first correctly 7/100 of 81/100 or (7 × 81)/(100 × 100);
and both answer incorrectly 7/100 of 19/100 or (7 × 19)/(100 × 100);[4]

4.11 Now suppose that on some question both give the same answer. The questions are always such as are to be answered *yes* or *no*. So the questions on which their answers agree are the same as those which both answer correctly together with those which both answer falsely, or (93 × 81)/(100 × 100) + (7 × 19)/(100 × 100) of all. The proportion of those which both answer correctly, out of those on which their answers agree, is therefore—

4.12 [(93 × 81)/(100 × 100)] / [(93 × 81)/(100 × 100) + (7 × 19)/(100 × 100)] or (93 × 81)/((93 × 81) + (7 × 19))

4.13 This, therefore, is the probability that, if both modes of inference yield the same result, that result is correct. Here we

may conveniently use another mode of expression. *Probability* is the ratio of the favorable cases to all the cases. Instead of expressing our result in terms of this ratio, we may use another: the ratio of favorable to unfavorable cases. This last ratio may be called the *chance* of an event.[5] Then the chance of a true answer by the first mode of inference is 81/19, and by the second is 93/7; and the chance of a correct answer from both, when they agree, is— (81 × 93)/(19 × 7) or 81/19 × 93/7, or the product of the chances of each singly yielding a true answer.

4.14 It will be seen that a chance is a quantity that may have any magnitude, however great. An event in whose favor there is an even chance, or 1/1, has a probability of 1/2. An argument having an even chance can do nothing toward reinforcing others, since by the rule its combination with another would only multiply the chance of the latter by 1.

4.15 Probability and chance undoubtedly belong primarily to consequences and are relative to premises. Nevertheless, we may speak of the chance of an event absolutely, meaning the chance of the combination of all the arguments concerning it that exist for us in the given state of our knowledge. Taken in this sense, it is incontestable that the chance of an event is intimately connected with the degree of our belief in it. Belief is certainly something more than a mere feeling; yet there is a feeling of believing, and this feeling does and ought to vary with the chance of the thing believed, as deduced from all the arguments. It would seem, therefore, that any quantity that varies with the chance might serve as a thermometer for the proper intensity of belief. Among all such quantities there is one that is peculiarly appropriate. When there is a very great chance, the feeling of belief ought to be very intense. Absolute certainty, or an infinite chance, can never be attained by mortals, and this may appropriately be represented by an infinite belief. As the chance diminishes, the feeling of believing should diminish,

4 The printed text reads “both answer correctly” in the last line, an evident slip for “incorrectly.”

5 Peirce’s “chance” is what is now usually called the odds.

until an even chance is reached. There it should vanish completely and incline neither toward nor away from the proposition. When the chance becomes smaller still, a contrary belief should spring up and should increase in intensity as the chance diminishes. And as the chance almost vanishes (which it can never quite do), the contrary belief should tend toward an infinite intensity. Now, there is one quantity that fulfills these conditions more simply than any other: the *logarithm* of the chance. But there is another consideration which, if admitted, must fix us to this choice for our thermometer. It is that our belief ought to be proportional to the weight of evidence, in this sense: two arguments that are entirely independent, neither weakening nor strengthening each other, ought, when they concur, to produce a belief equal to the sum of the intensities of belief that each would produce separately. Now, we have seen that the chances of independent concurrent arguments are to be multiplied together to get the chance of their combination. Therefore the quantities that best express the intensities of belief should be such that they are to be *added* when the *chances* are multiplied, in order to produce the quantity that corresponds to the combined chance. And the logarithm is the only quantity that fulfills this condition. There is a general law of sensibility called Fechner's psychophysical law. It states that the intensity of any sensation is proportional to the logarithm of the external force that produces it. It is entirely in harmony with this law that the feeling of belief should be as the logarithm of the chance, the chance being the expression of the state of facts that produces the belief.[6]

4.16 The rule for combining independent concurrent arguments takes a very simple form when it is expressed in terms of the intensity of belief, measured in the proposed way. It is this: take the sum of all the feelings of belief that would be produced separately by all the arguments *pro*, subtract from it the similar sum for the arguments *con*, and the remainder is the feeling of

6 Gustav Theodor Fechner (1801–1887), German founder of psychophysics; the law appears in his *Elements of Psychophysics* (1860).

belief we ought to have on the whole. This is a procedure men often resort to, under the name of *balancing reasons*.

4.17 These considerations make up an argument in favor of the conceptualistic view. Its kernel is that the joint probability of all the arguments in our possession concerning any fact must be intimately connected with the just degree of our belief in that fact. This point is supplemented by various others showing that the theory is consistent with itself and with the rest of our knowledge.

4.18 But probability, to have any value at all, must express a fact. It is therefore a thing to be inferred from evidence. Let us, then, consider for a moment how a belief about probability is formed. Suppose we have a large bag of beans, from which one has been secretly taken at random and hidden under a thimble. We are now to form a probable judgment of the color of that bean by drawing others singly from the bag and looking at them. Each one is to be thrown back, and the whole well mixed up after each drawing. Suppose the first drawing is white and the next black. We conclude that there is no immense preponderance of either color, and that there is something like an even chance that the bean under the thimble is black. But this judgment may be altered by the next few drawings. When we have drawn ten times, if 4, 5, or 6 are white, we have more confidence that the chance is even. When we have drawn a thousand times, if about half have been white, we have great confidence in this result. We now feel pretty sure that, if we were to make a large number of bets on the color of single beans drawn from the bag, we could approximately insure ourselves in the long run by betting each time on the white. That confidence would be entirely lacking if we had sampled the bag with only two drawings instead of 1,000. Now, the whole utility of probability is to insure us in the long run, and that assurance depends not merely on the value of the chance but also on the accuracy of its evaluation. It follows that we ought not to have the same feeling of belief about all events whose chance is even. In short, to express the proper state of our belief, not *one* number but *two* are

required: the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based.[7] It is true that when our knowledge is very precise, the second number may become insignificant, or vanish entirely. That number expresses the uncertainty of the assumed probability and its liability to be changed by further experience. Our knowledge is precise in this way when we have made many drawings from the bag, or when, as in most of the examples in the books, the total contents of the bag are absolutely known. But when our knowledge is very slight, this number may be even more important than the probability itself. And when we have no knowledge at all, it completely overwhelms the other. In that case there is no sense in saying that the chance of the totally unknown event is even (for what expresses absolutely no fact has absolutely no meaning), and what ought to be said is that the chance is entirely indefinite. We thus see that the conceptualistic view, though it works well enough in some cases, is quite inadequate.

4.19 Suppose that the first bean we drew from our bag were black. That would be an argument, however slender, that the bean under the thimble was black too. If the second bean also turned out black, that would be a second, independent argument reinforcing the first. If all of the first twenty beans drawn proved black, our confidence that the hidden bean was black would rightly become quite strong. But suppose the twenty-first bean were white, and that we went on drawing until we found we had drawn 1,010 black beans and 990 white ones. We should conclude that our first twenty beans coming up black was simply an extraordinary accident, that in fact the proportion of white beans to black was practically equal, and that it was an even chance that the hidden bean was black. Yet according to the rule of *balancing reasons*, since every drawing of a black bean is one more independent argument that the bean under the thimble is black, and every white drawing one more against it,

7 PEIRCE'S NOTE. Strictly, we should need an infinite series of numbers, each depending on the probable error of the one before.

an excess of twenty black beans ought to produce the same degree of belief that the hidden bean was black, whatever the total number drawn.

4.20 In the conceptualistic view of probability, complete ignorance, where the judgment ought to lean neither toward the hypothesis nor away from it, is represented by the probability 1/2.[8]

4.21 But suppose we are totally ignorant of what color hair the inhabitants of Saturn have. Let us, then, take a color chart in which all possible colors are shown shading into one another by imperceptible degrees. In such a chart the relative areas taken up by different classes of colors are perfectly arbitrary. Let us enclose one such area with a closed line and ask: what, on conceptualistic principles, is the chance that the hair color of the inhabitants of Saturn falls within that area? The answer cannot be indeterminate, because we must be in some state of belief; and, in fact, conceptualistic writers do not admit indeterminate probabilities. Since there is no certainty in the matter, the answer lies between *zero* and *unity*. Since the data supply no numerical value, the number must be fixed by the nature of the scale of probability itself, not by calculation from the data. The answer can therefore only be one-half, since the judgment should neither favor nor oppose the hypothesis. What is true of this area is true of any other; and it will be equally true of a third area that takes in the other two. But if the probability for each of the smaller areas is one-half, the probability for the larger should be at least unity, which is absurd.

EXPLICATIVE AND AMPLIFIATIVE REASONING

Probabilities cannot be assigned to synthetic conclusions; the urn argument from equal ignorance is absurd.

4.22 All our reasonings are of two kinds: 1. *Explicative, analytic*, or *deductive*; 2. *Amplifiative, synthetic*, or (loosely speaking)

8 PEIRCE'S NOTE. "Perfect indecision, belief inclining neither way, an even chance."—De Morgan, p. 182.

inductive. In explicative reasoning, certain facts are first laid down in the premises. These facts are, in every case, an inexhaustible multitude, but they can often be summed up in one simple proposition by means of some regularity running through them all. Take, for instance, the proposition that Socrates was a man. This implies (to go no further) that during every fraction of a second of his whole life (or, if you prefer, during most of them) he was a man. He did not appear at one instant as a tree and at another as a dog; he did not flow into water, or appear in two places at once; you could not put your finger through him as if he were an optical image; and so on. Now, with the facts laid down in this way, we may perhaps discover some order among some of them that was not particularly used in stating them; and this will let us recast part or all of them in a new statement, whose possibility might have escaped notice. Such a statement will be the conclusion of an analytic inference. All mathematical demonstrations are of this sort. But synthetic reasoning is of another kind. In this case the facts summed up in the conclusion are not among those stated in the premises. They are different facts—as when someone sees the tide rise *m* times and concludes that it will rise the next time. These are the only inferences that increase our real knowledge, however useful the others may be.

4.23 In any problem in probabilities, we are given the relative frequency of certain events, and we see that the relative frequency of another event is given, in a hidden way, in these facts. Stating it constitutes the solution. This is therefore mere explicative reasoning, and it is plainly quite inadequate to represent synthetic reasoning, which goes beyond the facts given in the premises. It is therefore manifestly impossible to work out a probability for a synthetic conclusion in this way.

4.24 Most treatises on probability contain a very different doctrine. They state, for example, that if one of the ancient inhabitants of the shores of the Mediterranean, who had never heard of tides, had gone to the Bay of Biscay and seen the tide rise there, say, *m* times, he could know that there was a probability equal

to $(m + 1)/(m + 2)$ that it would rise the next time. In a well-known work by Quetelet, much weight is placed on this, and it is made the foundation of a theory of inductive reasoning.[9]

4.25 But this solution betrays its origin if we apply it to the case in which the man has never seen the tide rise at all—that is, if we put $m = 0$. In that case the probability that it will rise the next time comes out 1/2; in other words, the solution involves the conceptualistic principle that a totally unknown event has an even chance. The result was reached by considering a number of urns, all containing the same number of balls, some white and some black. One urn contains all white balls, another one black and the rest white, a third two black and the rest white, and so on, one urn for each proportion, until we reach an urn containing only black balls. But the only possible reason for drawing any analogy between such an arrangement and the arrangement of Nature is the principle that alternatives of which we know nothing must be considered equally probable. And this principle is absurd. There are countless ways of enumerating the different possibilities, and applying this principle to them would give different results. If there is any way of enumerating the possibilities so as to make them all equal, it is not the one this solution is derived from, but the following. Suppose we had an immense granary filled with black and white balls, well mixed; and suppose each urn were filled by taking a fixed number of balls from this granary entirely at random. The relative number of white balls in the granary might be anything—say one in three. Then in one-third of the urns the first ball would be white, and in two-thirds black. In one-third of the urns whose first ball was white, and also in one-third of those whose first ball was black, the second ball would be white. In this way we should get a distribution like the one shown in the following table, where *w* stands for a white ball and *b* for a black one. The reader can, if he likes, verify the table for himself.

9 The formula is Laplace's "rule of succession."

4.26 wwww.
wwwb. wwbw. wbww. bwww.
wwwb. wwbw. wbww. bwww.
wwbb. wbwb. bwwb. wbbw. bwbw. bbww.
wwbb. wbwb. bwwb. wbbw. bwbw. bbww.
wwbb. wbwb. bwwb. wbbw. bwbw. bbww.
wwbb. wbwb. bwwb. wbbw. bwbw. bbww.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.
bbbb.

4.27 In the second group, where there is one b, there are two identical sets; in the third there are 4, in the fourth 8, and in the fifth 16, doubling each time. This is because we have supposed twice as many black balls in the granary as white ones. Had we supposed 10 times as many, then instead of 1, 2, 4, 8, 16 sets we

should have had 1, 10, 100, 1000, 10000 sets. On the other hand, had the numbers of black and white balls in the granary been equal, there would have been only one set in each group. Now suppose two balls were drawn from one of these urns and both were found to be white: what would be the probability of the next one being white? If the two drawn were the first two put into the urn, and the next to be drawn were the third put in, then the probability of this third ball being white would be the same whatever the colors of the first two. For we have supposed that exactly the same proportion of urns has the third ball white among those whose first two are *white-white, white-black, black-white,* and *black-black.* So in this case the chance of the third ball being white would be the same whatever the first two were. But by inspecting the table the reader can see that in each group all orders of the balls occur equally often, so that it makes no difference whether they are drawn out in the order they were put in or not. Hence the colors of the balls already drawn have no influence on the probability of any other being white or black.

4.28 Now, if there is any way of enumerating the possibilities of Nature so as to make them equally probable, it is clearly one that would make any one arrangement or combination of the elements of Nature as probable as any other—that is, a distribution like the one we have supposed. It therefore appears that the assumption that anything of the kind can be done leads simply to the conclusion that reasoning from past to future experience is absolutely worthless. In fact, the moment you assume that the chances in favor of what we are totally ignorant of are even, the problem about the tides does not differ in any arithmetical respect from the case of a penny (known to be equally likely to come up heads or tails) turning up heads *m* times in a row. In short, it would be to assume that Nature is a pure chaos, a chance combination of independent elements, in which reasoning from one fact to another would be impossible. And since, as we shall see later, there is no judgment of pure observation without reasoning, it would be to suppose all human cognition illusory and no real knowledge possible. It would be to suppose

that if we have found the order of Nature more or less regular in the past, this has been a pure run of luck, which we may expect has now come to an end. Now, it may be that we have not a scintilla of proof to the contrary; but reasons are unnecessary for the belief that is the most settled of all, which nobody doubts or can doubt, and which anyone who denied it would reduce himself to absurdity by denying.

4.29 The relative probability of this or that arrangement of Nature is something we should have a right to talk about if universes were as plentiful as blackberries—if we could put a quantity of them in a bag, shake them up well, draw out a sample, and examine them to see what proportion had one arrangement and what proportion another. But even in that case a higher universe would contain us, and to its arrangements the conception of probability could have no application.

HOW OFTEN INDUCTION IS RIGHT

We can know how often inferences from samples come out right, as the census example shows.

4.30 We have examined the problem proposed by the conceptualists, which, put into clear language, is this: Given a synthetic conclusion, required to know how many of all possible states of things will agree, to any assigned extent, with this conclusion. And we have found that it is only an absurd attempt to reduce synthetic to analytic reasoning, and that no definite solution is possible.

4.31 But there is another problem connected with this subject. It is this: Given a certain state of things, required to know what proportion of all synthetic inferences relating to it will be true within a given degree of approximation. Now, there is no difficulty about this problem (apart from its mathematical complication); it has been much studied, and the answer is perfectly well known. And is this not, after all, what we want to know, much more than the other? Why should we want to know the probability that the fact will agree with our conclusion? That implies that

we are interested in all possible worlds, and not merely the one in which we find ourselves. Why is it not much more to the purpose to know the probability that our conclusion will agree with the fact? One of these questions is the first stated above and the other the second. And I ask the reader whether, if people had always spoken of relative frequency instead of using the word probability with no clear grasp of what they meant by it, they could have failed to see what they wanted. It was not to follow the synthetic procedure with an analytic one in order to find the probability of the conclusion; on the contrary, it was to begin with the fact at which the synthetic inference aims, and trace back to the facts it uses as premises, in order to see the probability of their being such as will yield the truth.

4.32 Since we cannot have an urn with an infinite number of balls to represent the inexhaustibleness of Nature, let us suppose one with a finite number, each ball being thrown back into the urn after it is drawn, so that they are never exhausted. Suppose one ball in three is white and the rest black, and that four balls are drawn. Then the table on page 713 shows the relative frequency of the different ways in which these balls might be drawn.[10] It will be seen that if we judged the proportion in the urn by these four balls, 32 times out of 81 we should find it 1/4, and 24 times out of 81 we should find it 1/2, the truth being 1/3. Extending this table to high numbers would be a great labor, but mathematicians have found some ingenious ways of calculating what the numbers would be. It turns out that if the true proportion of white balls is p, and s balls are drawn, then the error of the proportion obtained by the induction will be—

10 The page number is that of the original magazine printing (Popular Science Monthly); the table is not reproduced here.

4.33 half the time within 0.477 √(2p(1 − p)/s)
9 times out of 10 within 1.163 √(2p(1 − p)/s)
99 times out of 100 within 1.821 √(2p(1 − p)/s)
999 times out of 1,000 within 2.328 √(2p(1 − p)/s)
9,999 times out of 10,000 within 2.751 √(2p(1 − p)/s)
9,999,999,999 times out of 10,000,000,000 within 4.77 √(2p(1 − p)/s)

4.34 An example will show how this is used. According to the census of 1870, the proportion of males among native white children under one year old was 0.5082, while among Black children of the same age it was only 0.4977. The difference between these is 0.0105, or about one in 100. Can this be put down to chance, or would the difference always exist among a large number of white and Black children in similar circumstances? Here *p* may be taken as 1/2; hence 2*p* (1-*p*) is also 1/2. The number of white children counted was nearly 1,000,000; so the fraction whose square root is to be taken is about 1/2000000. The root is about 1/1400, and this multiplied by 0.477 gives about 0.0003 as the probable error in the ratio of males among the white children as obtained by the induction. The number of Black children was about 150,000, which gives 0.0008 for the probable error. We see that the actual discrepancy is ten times the sum of these, and according to our table such a result would happen only once in 10,000,000,000 censuses, in the long run.

4.35 It may be noted that when the real value of the probability sought by induction is either very large or very small, the reasoning is more secure. Suppose, for example, that there were in reality one white ball in 100 in a certain urn, and we were to judge the number by 100 drawings. The probability of drawing no white ball would be 366/1000; of drawing one white ball, 370/1000; of drawing two, 185/1000; of drawing three, 61/1000; of drawing four, 15/1000; of drawing five, only 3/1000; and so on. So we should be fairly certain of not being in error by more than one ball in 100.

4.36 It appears, then, that in one sense we can, and in another we cannot, determine the probability of synthetic inference. When I reason like this:

4.37 Ninety-nine Cretans in a hundred are liars;
But Epimenides is a Cretan;
Therefore, Epimenides is a liar:—[11]

4.38 I know that reasoning like that would lead to the truth 99 times in 100. But when I reason in the opposite direction:

4.39 Minos, Sarpedon, Rhadamanthus, Deucalion, and
Epimenides are all the Cretans I can think of;
But these were all atrocious liars;
Therefore, pretty much all Cretans must have been liars;[12]

4.40 I have not the slightest idea how often such reasoning would lead me right. What I do know, on the other hand, is that some definite proportion of Cretans must have been liars, and that this proportion can probably be approximated by an induction from five or six instances. Even in the worst case for the probability of such an inference—the case in which about half the Cretans are liars—the ratio so obtained would probably not be in error by more than 1/6. That much I know. But in the present case the inference is that pretty much all Cretans are liars, and whether there may be a special improbability in that, I do not know.

HOW SYNTHETIC JUDGMENTS ARE POSSIBLE

Extending Kant's question, Peirce grounds induction in sampling: experiences under the same conditions share characters.

4.41 Late in the eighteenth century, Immanuel Kant asked the question, "How are synthetical judgments *a priori* possible?" By synthetical judgments he meant those that assert positive fact and are not mere matters of arrangement—in short, judgments

11 Epimenides, a semi-legendary Cretan seer, is credited with the line "Cretans are always liars," the source of the liar paradox.
12 Minos, Sarpedon, Rhadamanthus and Deucalion are figures of Cretan legend.

of the kind that synthetical reasoning produces and analytic reasoning cannot yield. By *a priori* judgments he meant such judgments as that all outward objects are in space, that every event has a cause, and so on—propositions which, according to him, can never be inferred from experience. Not so much by his answer to this question as by the mere asking of it, the current philosophy of the time was shattered and destroyed, and a new epoch in its history began.[13] But before asking *that* question he ought to have asked the more general one: "How are any synthetical judgments at all possible?" How is it that a man can observe one fact and immediately pronounce judgment on another, different fact not involved in the first? Such reasoning, as we have seen, has no definite probability, at least in the usual sense of the phrase; how, then, can it add to our knowledge? This is a strange paradox. The Abbé Gratry says it is a miracle, and that every true induction is an immediate inspiration from on high.[14] I respect this explanation far more than many a pedantic attempt to solve the question by some juggling with probabilities, with the forms of syllogism, or whatever else.[15] I respect it because it shows an appreciation of the depth of the problem, because it assigns an adequate cause, and because it is intimately connected—as the true account should be—with a general philosophy of the universe. At the same time, I do not accept this explanation, because an explanation should tell *how* a thing is done, and to assert a perpetual miracle seems to be giving up all hope of doing that, without sufficient justification.

4.42 It will be interesting to see how the answer Kant gave to his question about synthetical judgments *a priori* looks when it is extended to the question of synthetical judgments in general. That answer is that synthetical judgments *a priori* are possible because whatever is universally true is involved in the condi-

13 Immanuel Kant (1724–1804), German philosopher; he posed the question in the *Critique of Pure Reason* (1781).

14 PEIRCE'S NOTE. *Logique*. According to him, the same is true of every performance of a differentiation, but not of integration. He does not tell us whether it is the supernatural assistance that makes the former process so much the easier.

15 Auguste Gratry (1805–1872), French priest, philosopher and mathematician.

tions of experience. Let us apply this to a general synthetical reasoning. I take a handful of beans from a bag; they are all purple, and I infer that all the beans in the bag are purple. How can I do that? Why, on the principle that whatever is universally true of my experience (which here is the appearance of these different beans) is involved in the condition of experience. The condition of this particular experience is that all these beans were taken from that bag. According to Kant's principle, then, whatever is found true of all the beans drawn from the bag must find its explanation in some peculiarity of the contents of the bag. This is a satisfactory statement of the principle of induction.

4.43 When we draw a deductive or analytic conclusion, our rule of inference is that facts of a certain general character are accompanied, either invariably or in a certain proportion of cases, by facts of another general character. Then, our premise being a fact of the first class, we infer with certainty, or with the appropriate degree of probability, the existence of a fact of the second class. But the rule for synthetic inference is of a different kind. When we sample a bag of beans, we do not in the least assume that the fact of some beans being purple involves the necessity, or even the probability, of other beans being purple. On the contrary, the conceptualistic method of treating probabilities, which really amounts simply to treating them deductively, leads, when rightly carried out, to the result that a synthetic inference has just an even chance in its favor—in other words, that it is absolutely worthless. The color of one bean is entirely independent of that of another. But synthetic inference is founded on a classification of facts, not according to their characters, but according to the way they are obtained. Its rule is that a number of facts obtained in a given way will in general more or less resemble other facts obtained in the same way; or, *experiences whose conditions are the same will have the same general characters*.

4.44 In the former case, we know that premises exactly similar in form to the given ones will yield true conclusions just once in

a calculable number of times. In the latter case, we know only that premises obtained under circumstances similar to the given ones (though perhaps themselves very different) will yield true conclusions at least once in a calculable number of times. We may put this by saying that in the case of analytic inference we know the probability of our conclusion (if the premises are true), but in the case of synthetic inferences we know only the degree of trustworthiness of our procedure. Since all knowledge comes from synthetic inference, we must equally infer that all human certainty consists merely in our knowing that the processes by which our knowledge has been derived are such as must generally have led to true conclusions.

4.45 Although a synthetic inference cannot by any means be reduced to deduction, the fact that the rule of induction will hold good in the long run can be deduced from the principle that reality is only the object of the final opinion to which sufficient investigation would lead. That belief gradually tends to fix itself under the influence of inquiry is, indeed, one of the facts logic starts from.

The Order of Nature

Is the universe ordered? Chance, law and design, and why the order we find tells us little about a designer.

THE ORDER OF NATURE I.

5.1 Any proposition whatever about the order of Nature must touch more or less on religion. In our day, belief, even in these matters, depends more and more on the observation of facts. If a remarkable and universal orderliness is found in the universe, there must be some cause for this regularity, and science has to consider what hypotheses might account for the phenomenon. One way of accounting for it, certainly, would be to suppose that the world is ordered by a superior power. But if there is nothing in the universal subjection of phenomena to laws, nor in the character of those laws themselves (their being benevolent, beautiful, economical, etc.), that goes to prove the existence of a governor of the universe, it is hardly to be expected that any other kind of evidence will carry much weight with minds freed from the tyranny of tradition.

5.2 Nevertheless, it cannot truly be said that even an absolutely negative answer to that question could destroy religion altogether. For there are faiths in which, however much they differ from our own, we recognize the essential characters that make them worthy to be called religions, and which nevertheless do not postulate an actually existing Deity. The one, for instance, that has had the largest—and by no means the least intelligent—following of any on earth teaches that the Divinity in his highest perfection is withdrawn from the world in a state of profound and eternal sleep, which really does not differ from non-exist-

ence, whether it is called by that name or not.[1] No fair-minded person who has followed the writings of M. Vacherot can deny that his religion is as earnest as can be. He worships the Perfect, the Supreme Ideal; but he holds that the very notion of the Ideal is incompatible with its real existence. In fact, M. Vacherot finds it agreeable to his reason to assert that non-existence is an essential character of the perfect, just as St. Anselm and Descartes found it agreeable to theirs to assert the extreme opposite.[2] I confess that in one respect either of these positions seems to me more in keeping with the religious attitude than that of a theology which rests on evidences. For as soon as the Deity presents himself to either Anselm or Vacherot and shows his glorious attributes, whether in a vision of the night or of the day, either of them recognizes his adorable God and sinks to his knees at once. The theologian of evidences, by contrast, will first demand that the divine apparition identify himself; and only after scrutinizing his credentials and weighing the probabilities of his being found among the totality of existing things will he finally pay his cautious homage, thinking that no characters can be adorable except those that belong to a real thing.

5.3 If we could find any general characteristic of the universe, any mannerism in the ways of Nature, any law applicable everywhere and universally valid, such a discovery would be of such singular help to us in all our future reasoning that it would deserve a place almost at the head of the principles of logic. On the other hand, if it can be shown that there is nothing of the sort to find, and that every discoverable regularity is of limited range, this again will be of logical importance. What sort of conception we ought to have of the universe, how to think of the *ensemble* of things, is a fundamental problem in the theory of reasoning.

1 Peirce appears to mean Buddhism, with nirvana understood in the manner of nineteenth-century Western interpreters.

2 Étienne Vacherot (1809–1897), French philosopher. Anselm and Descartes gave versions of the ontological argument, that perfection implies existence.

IS THE WORLD A CHANCE-WORLD?

A world without laws would be more orderly than ours, and the idea of a chance-world is contradictory.

5.4 It is the legitimate endeavor of scientists now, as it was twenty-three hundred years ago, to account for the formation of the solar system, and of the cluster of stars that forms the galaxy, by the fortuitous concourse of atoms. The greatest expounder of this theory, when asked how he could write an immense book on the system of the world without once mentioning its author, replied, very logically, "I had no need of that hypothesis." But in truth there is nothing atheistic in the theory, any more than there was in this answer.[3] Matter is supposed to be composed of molecules that obey the laws of mechanics and exert certain attractions on one another; and it is to these regularities (which no attempt is made to account for), and not to hazard, that the general arrangement of the solar system would be due.

5.5 If anyone has ever maintained that the universe is a pure throw of the dice, the theologians have abundantly refuted him. "How often," says Archbishop Tillotson, "might a man, after he had jumbled a set of letters in a bag, fling them out upon the ground before they would fall into an exact poem, yea, or so much as make a good discourse in prose![4] And may not a little book be as easily made by chance as this great volume of the world?" The chance-world shown here to be so different from the one we live in would be one without laws, in which the characters of different things were entirely independent; so that if a sample of any kind of object ever showed a prevalent character, it could only be by accident, and no general proposition could ever be established. Whatever further conclusions we may

3 The ancient atomists were Democritus and Epicurus. The reply is attributed to Pierre-Simon Laplace, speaking to Napoleon about his Mécanique céleste; original: "Je n'avais pas besoin de cette hypothèse-là."

4 John Tillotson (1630–1694), Archbishop of Canterbury, from his sermon "The Wisdom of Being Religious."

reach about the order of the universe, this much may be regarded as solidly established: the world is not a mere chance-medley.

5.6 But whether the world makes an exact poem or not is another question. When we look up at the sky at night, we readily see that the stars are not simply splashed onto the celestial vault; but there does not seem to be any precise system in their arrangement either. It will be worth our while, then, to inquire into the degree of orderliness in the universe; and, to begin with, let us ask whether the world we live in is any more orderly than a purely chance-world would be.

5.7 Any uniformity, or law of Nature, may be stated in the form, "Every A is B"; for example, every ray of light is a non-curved line, every body is accelerated toward the earth's center, etc. This is the same as saying, "There does not exist any A that is not B": there is no curved ray; there is no body not accelerated toward the earth. So the uniformity consists in the non-occurrence in Nature of a certain combination of characters (in this case, the combination of being A with being non-B).[5] And, conversely, every case of the non-occurrence of a combination of characters would constitute a uniformity in Nature. Thus, suppose the quality A is never found combined with the quality C: for example, suppose the quality of severe intellectual disability is never found combined with that of having a well-developed brain. Then nothing of the sort A is of the sort C, or everything of the sort A is of the sort non-C (or, say, every person with a severe intellectual disability has an ill-developed brain); and this, being something universally true of the A's, is a uniformity in the world. We see, then, that in a world with no uniformities no logically possible combination of characters would be excluded, but every combination would exist in some object. But two objects that are not identical must differ in some of their

5 PEIRCE'S NOTE. For the present purpose, the negative of a character is to be considered as much a character as the positive, since a uniformity may be either affirmative or negative. I do not say that no distinction can be drawn between positive and negative uniformities.

characters, even if only in the character of being in such-and-such a place. Hence exactly the same combination of characters could not be found in two different objects; and consequently, in a chance-world, every combination involving either the positive or the negative of every character would belong to just one thing. So if there were only five simple characters in such a world,[6] we might denote them by A, B, C, D, E, and their negatives by a, b, c, d, e. Then, since there would be 2^5 or 32 different combinations of these characters, each completely determinate with respect to every one of them, that world would contain just 32 objects, with characters as in the following table:

5.8 Table I.

ABCDE AbCDE aBCDE abCDE
ABCDe AbCDe aBCDe abCDe
ABCdE AbCdE aBCdE abCdE
ABCde AbCde aBCde abCde
ABcDE AbcDE aBcDE abcDE
ABcDe AbcDe aBcDe abcDe
ABcdE AbcdE aBcdE abcdE
ABcde Abcde aBcde abcde

5.9 For example, if the five primary characters were *hard, sweet, fragrant, green, bright,* there would be one object that combined all these qualities; one that was hard, sweet, fragrant, and green, but not bright; one that was hard, sweet, fragrant, and bright, but not green; one that was hard, sweet, and fragrant, but neither green nor bright; and so on through all the combinations.

5.10 This is what a thoroughly chance-world would be like, and certainly nothing more systematic could be imagined. When a quantity of letters is poured out of a bag, the appearance of disorder is due to the fact that the phenomena are only partly fortuitous. In that case the laws of space are supposed to be rigidly maintained, and there is also a certain amount of regularity in

6 PEIRCE'S NOTE. With 5 simple characters and their negatives, they could be compounded in various ways to make 241 characters in all, not counting the characters *existence* and *non-existence,* which bring the total to 243, or 3^5.

the formation of the letters. The result is that some elements are orderly and some disorderly, which is precisely what we observe in the actual world. Tillotson, in the passage part of which has been quoted, goes on to ask, "How long might 20,000 blind men, which should be sent out from the several remote parts of England, wander up and down before they would all meet upon Salisbury Plains, and fall into rank and file in the exact order of an army? And yet this is much more easy to be imagined than how the innumerable blind parts of matter should rendezvous themselves into a world." This is very true; but in the actual world the *blind men* are, as far as we can see, *not* drawn up in any particular order at all. In short, while a certain amount of order exists in the world, it seems that the world is not as orderly as it might be—not, for instance, as orderly as a world of pure chance would be.

5.11 But we can never get to the bottom of this question until we take account of a highly important logical principle[7], which I will now state. This principle is that any plurality or lot of objects whatever has some character in common (no matter how insignificant) that is peculiar to them and not shared by anything else. The word "character" is taken here in a sense that includes negative characters, such as incivility, inequality, etc., as well as their positives, civility, equality, etc. To prove the theorem, I will show what character any two things, A and B, have in common that is not shared by anything else. The things A and B are each distinguished from all other things by having certain characters, which may be named A-ness and B-ness. Corresponding to these positive characters are the negative characters un-A-ness, which everything except A possesses, and un-B-ness, which everything except B possesses. These two characters are united in everything except A and B. This union of the characters un-A-ness and un-B-ness makes a compound character, which may be called A-B-lessness. Neither A nor B possesses it, but everything else does. This character, like every other, has its corresponding negative, un-A-B-lessness, and this last is the

7 PEIRCE'S NOTE. This principle was, I believe, first stated by Mr. De Morgan.

character possessed by both A and B and by nothing else. It is obvious that what has just been shown true of two things is, *mutatis mutandis*, true of any number of things. Q.E.D.[8]

5.12 In any world whatever, then, there must be a character peculiar to each possible group of objects. If, as a matter of naming, characters peculiar to the same group are regarded as only different aspects of the same character, then we may say that there will be exactly one character for each possible group of objects. Suppose, for example, that a world contains five things, α, β, γ, δ, ε. Then it will have a separate character for each of the 31 groups (with *non-existence* making up 32, or 2^5) shown in the following table:

5.13 Table II.

α, β, γ, δ, ε
αβ, αγ, αδ, αε, βγ, βδ, βε, γδ, γε, δε
αβγ, αβδ, αβε, αγδ, αγε, αδε, βγδ, βγε, βδε, γδε
αβγδ, αβγε, αβδε, αγδε, βγδε
αβγδε

5.14 This shows that the very idea of a chance-world involves a contradiction. In a world of 32 things, instead of there being only 3^5, or 243, characters, as we have seen the notion of a chance-world requires, there would in fact be no fewer than 2^{32}, or 4,294,967,296, characters. These would not all be independent, but would have every possible relation to one another.

5.15 We further see that as long as we regard characters abstractly, without regard to their relative importance and so on, there is no possibility of a greater or lesser degree of orderliness in the world. The whole system of relationships between the different characters is given by mere logic; that is, it is implied in those facts which are tacitly admitted as soon as we admit that there is any such thing as reasoning.

5.16 To come down from this abstract point of view, we have to consider the characters of things as relative to the perceptions

8 *Mutatis mutandis*: "with the necessary changes made." Q.E.D. (*quod erat demonstrandum*): "which was to be proved."

and active powers of living beings. Instead, then, of trying to imagine a world in which there were no uniformities, let us suppose one in which none of the uniformities had reference to characters that are interesting or important to us. In the first place, there would be nothing to puzzle us in such a world. The small number of qualities that would directly meet the senses would be the ones that gave the key to everything that could possibly interest us. The whole universe would have such an air of system and perfect regularity that there would be nothing to ask. In the next place, no action of ours, and no event of Nature, would have important consequences in such a world. We would be perfectly free of all responsibility, and there would be nothing to do but enjoy or suffer whatever happened to come along. So there would be nothing to stimulate or develop either the mind or the will, and consequently we would neither act nor think. We would have no memory, because memory depends on a law of our organization. Even if we had any senses, we would stand toward such a world exactly as inanimate objects stand toward the present one—provided we suppose that these objects have an absolutely transitory and instantaneous consciousness without memory, a supposition that is a mere manner of speaking, since that would be no consciousness at all. We may say, therefore, that a world of chance is simply our actual world seen from the standpoint of an animal at the very vanishing point of intelligence. To the mind of a polyp, the actual world is almost a chance-medley. The interest that the uniformities of Nature have for an animal measures his place on the scale of intelligence.

5.17 Thus nothing can be made out from the orderliness of Nature about the existence of a God, unless it is maintained that the existence of a finite mind proves the existence of an infinite one.

INDUCTION AS SAMPLING

Induction concerns a character named beforehand in a random sample, not a uniformity of Nature as Mill held.

5.18 In the last of these papers we examined the nature of inductive or synthetic reasoning. We found it to be a process of sampling. A number of specimens of a class are taken, not by selection within that class, but at random. These specimens will agree in a great many respects. Now, if it were likely that a second lot would agree with the first in most of these respects, we might base on this consideration an inference about any one of these characters. But such an inference would not be of the nature of induction, nor would it (except in special cases) be valid, because the vast majority of the points of agreement in the first sample drawn would generally be entirely accidental, as well as insignificant. To illustrate this, I take the ages at death of the first five poets listed in Wheeler's "Biographical Dictionary." They are:

5.19 Aagard, 48.
Abeille, 70.
Abulola, 84.
Abunowas, 48.
Accords, 45.

5.20 These five ages have the following characters in common:

5.21 1. The difference of the two digits making up the number, divided by three, leaves a remainder of *one*.

5.22 2. The first digit, raised to the power indicated by the second and divided by three, leaves a remainder of *one*.

5.23 3. The sum of the prime factors of each age, including one, is divisible by three.

5.24 It is easy to see that there would be no end to accidental agreements of this sort. But suppose that, instead of considering a character because it prevails in the sample, we designate a character before taking the sample, choosing it for its importance, obviousness, or some other point of interest. Then two siz-

able samples drawn at random are extremely likely to agree approximately in the proportion of occurrences of a character chosen in this way. *The inference that a previously designated character has nearly the same frequency of occurrence in the whole of a class that it has in a sample drawn at random out of that class is induction.* If the character has not been designated beforehand, then a sample in which it is found to prevail can only serve to suggest that it *may be* prevalent in the whole class. We may call this surmise an inference if we like—an inference of possibility—but a second sample must be drawn to test whether the character actually is prevalent. Instead of designating beforehand a single character with reference to which we will examine a sample, we may designate two and use the same sample to determine the relative frequencies of both. This amounts to making two inductive inferences at once, and of course we are less certain that both will yield correct conclusions than we would be that either one separately would. What is true of two characters is true of any limited number. Now, the number of characters that have any considerable interest for us with reference to any class of objects is smaller than one might suppose. Since we are sure to examine any sample with reference to these characters, they may be regarded not exactly as predesignated but as predetermined (which amounts to the same thing). We may then, if we like, infer that the sample represents the class in all these respects—remembering only that this is not as secure an inference as it would be if the particular quality to be looked for had been fixed on beforehand.

5.25 The demonstration of this theory of induction rests on principles, and follows methods, that are accepted by everyone who shows in other matters the particular knowledge and force of mind that qualify one to judge of this. Yet the theory itself, quite unaccountably, seems never to have occurred to any of the writers who have undertaken to explain synthetic reasoning. The most widespread opinion on the matter is one that Mr. John Stuart Mill did much to promote—namely, that induction depends for its validity on the uniformity of Nature; that is, on

the principle that what happens once will, given a sufficient degree of similarity of circumstances, happen again as often as the same circumstances recur.[9] It is applied like this: the fact that different things belong to the same class constitutes the similarity of circumstances, and the induction is good provided this similarity is "sufficient." What happens once is that a number of these things are found to have a certain character; what may then be expected to happen again as often as the circumstances recur is that all things belonging to the same class should have the same character.

5.26 This analysis of induction has, I venture to think, various imperfections, and it may be useful to call attention to some of them. In the first place, when I put my hand in a bag and draw out a handful of beans, and, finding three-quarters of them black, infer that about three-quarters of all the beans in the bag are black, my inference is obviously of the same kind as if I had found some larger proportion of the sample, or the whole of it, black, and had assumed that it represented the rest of the bag's contents in that respect. But the analysis in question hardly seems suited to explaining this *proportionate* induction, where the conclusion, instead of being that a certain event happens uniformly under certain circumstances, is precisely that it does not happen uniformly, but only in a certain proportion of cases. True, the whole sample may be regarded as a single object, and the inference may be brought under the proposed formula by taking the conclusion to be that any similar sample will show a similar proportion among its constituents. But this is to treat the induction as if it rested on a single instance, which gives a very false idea of its probability.

5.27 In the second place, if the uniformity of Nature were the sole warrant of induction, we would have no right to draw one about a character whose constancy we knew nothing about. Accordingly, Mr. Mill says that, although for thousands of years Europeans knew of no swans but white ones, the inference that

9 John Stuart Mill (1806–1873), English philosopher, whose *System of Logic* (1843) was the standard account of induction.

all swans were white was "not a good induction," because it was not known that color was a usual generic character (and in fact it is by no means one). But it can be demonstrated mathematically that an inductive inference may have as high a degree of probability as you please, independently of any prior knowledge of the constancy of the character inferred. Before it was known that color is not usually a character of *genera*, there was certainly a considerable probability that all swans were white. But further study of the *genera* of animals led to the induction that they are not uniform in color. A deductive application of this general proposition would have gone far to overcome the probability of the universal whiteness of swans before the black species was discovered.[10] When we do know something about the general constancy or inconstancy of a character, applying that general knowledge to the particular class an induction concerns does increase or diminish the force of the induction; but, like every application of general knowledge to particular cases, it is deductive in nature, not inductive.

5.28 In the third place, to say that inductions are true because similar events happen in similar circumstances—or, what is the same thing, because objects similar in some respects are likely to be similar in others—is to overlook the conditions that really are essential to the validity of inductions. When we take all characters into account, any pair of objects resemble each other in just as many particulars as any other pair. If we limit ourselves to characters that have some importance, interest, or obviousness for us, then a synthetic conclusion may be drawn—but only on condition that the specimens by which we judge have been taken at random from the class we are to judge about, and not selected as belonging to some sub-class. The induction has its full force only when the character concerned has been designated before the sample is examined. These are the essentials of induction, and they go unrecognized when the validity of induction is attributed to the uniformity of Nature. The explanation of induction by the doctrine of probabilities, given in the last of

10 *Genera*: plural of *genus*, the biological rank above species.

these papers, is not a mere metaphysical formula; it is one from which all the rules of synthetic reasoning can be deduced systematically and with mathematical rigor. But accounting for the matter by a principle of Nature, even if it were satisfactory in other respects, has the fatal disadvantage of leaving us just as much at sea as before about the proper method of induction. It does not surprise me, therefore, that those who adopt this theory have given erroneous rules for the conduct of reasoning, nor that most of the examples put forward by Mr. Mill in his first edition as models of what inductions should be turned out, in the light of later scientific progress, so singularly unfortunate that they had to be replaced by others in later editions. One would have thought Mr. Mill might have based an induction on *this* circumstance, especially since it is his avowed principle that if the conclusion of an induction turns out false, it cannot have been a good induction. Nevertheless, neither he nor any of his followers seems to have been led in the least to doubt the perfect solidity of the framework he devised for securely supporting the mind in its passage from the known to the unknown—even though, at its first trial, it did not work quite as well as had been expected.

CAUSES, SPACE AND TIME

Every event has a cause, and our inborn ideas of space, time and force fit Nature remarkably well.

5.29 Whenever we have drawn a statistical induction—for instance, that one-half of all births are of male children—it is always possible, by sufficiently prolonged investigation, to discover a class of which the same predicate can be affirmed universally; to find out, for instance, *what sort of* births are of male children. The truth of this principle follows immediately from the theorem that there is a character peculiar to every possible group of objects. The form in which the principle is usually stated is that *every event must have a cause*.

5.30 But although every event has a cause, and one of a kind that is capable of being discovered, still, if there is nothing to guide us to the discovery—if we have to hunt among all the events in the world without any scent; if, for instance, the sex of a child could equally well be supposed to depend on the configuration of the planets, on what was going on at the antipodes, or on anything else—then the discovery would have no chance of ever being made.

5.31 We have no right to assume that we ever do discover the precise causes of things, or that any induction whatever is absolutely without exception. On the contrary, it is an easy corollary of the theorem just mentioned that every empirical rule has an exception. But some of our inductions approach universality so extraordinarily that, even if we are to suppose they are not strictly universal truths, we cannot possibly think they were reached merely by accident. The most remarkable laws of this kind are those of *time* and *space*. With regard to space, Bishop Berkeley was the first to show, very conclusively, that it is not a thing *seen* but a thing *inferred*.[11] Berkeley chiefly insists that it is impossible to see the third dimension of space directly, since the retina of the eye is a surface. But in point of fact the retina is not even a surface. It is a cluster of nerve-needles pointing toward the light, with only their extreme tips sensitive, and these tips lie at considerable distances from one another compared with their areas. Now, the excitation of no single one of these points can produce the perception of a surface, and so neither can the aggregate of all the sensations. But certain relations hold between the excitations of different nerve-points, and these constitute the premises on which the hypothesis of space is founded and from which it is inferred. It is now universally admitted that space is not immediately perceived; and a mediate cognition is what is called an inference, and is subject to the criticism of logic. But what are we to say to the fact that every chicken, as soon as it is hatched, solves a problem whose data

11 George Berkeley (1685–1753), Irish philosopher and bishop, in *An Essay towards a New Theory of Vision* (1709).

are complex enough to test the greatest mathematical powers? It would be insane to deny that the tendency to hit upon the conception of space is inborn in the mind of the chicken and of every animal. The same is equally true of time. That time is not directly perceived is evident, since no lapse of time is present, and we perceive only what is present. It will probably also be admitted that, if we did not have the idea of time, we could never perceive the flow in our sensations without some particular aptitude for it. The idea of force—at least in its rudiments—is another conception that is arrived at so early, and is found in animals so low on the scale of intelligence, that it must be supposed innate. But the innateness of an idea admits of degree, for it consists in the tendency of that idea to present itself to the mind. Some ideas, like that of space, present themselves irresistibly at the very dawn of intelligence and take possession of the mind on slight provocation, while toward other conceptions we are predisposed, it is true, but less strongly, down a scale that extends a long way. The tendency to personify everything and attribute human characters to it may be called innate; but civilized man very soon overcomes it with regard to most of the objects around him. Take a conception such as gravitation varying inversely as the square of the distance. It is a very simple law. But to say that it is simple is merely to say that the mind is particularly suited to grasp it with ease. Suppose the idea of one quantity multiplied by another had been no easier for the mind than the idea of a quantity raised to the power indicated by itself—would we ever have discovered the law of the solar system?

5.32 It seems beyond dispute, therefore, that the human mind is strongly adapted to comprehending the world—at least to this extent, that certain conceptions highly important for such comprehension arise naturally in it. Without such a tendency, the mind could never have developed at all.

5.33 How are we to explain this adaptation? The conceptions of time, space, and force are so useful and so indispensable, even to the lowest intelligence, as to suggest that they are the results of natural selection. Without something like geometrical, kin-

etic, and mechanical conceptions, no animal could seize his food or do anything that might be necessary for the preservation of the species. He might, it is true, be provided with an instinct that would generally have the same effect; that is, he might have conceptions different from those of time, space, and force, but coinciding with them in the ordinary cases of the animal's experience. But since an animal whose mechanical conceptions did not break down in a novel situation (such as development must bring about) would have an immense advantage in the struggle for life, there would be constant selection in favor of more and more correct ideas of these matters. In this way would be attained knowledge of the fundamental law on which all science turns: namely, that forces depend on relations of time, space, and mass. Once this idea was sufficiently clear, no more than an understandable degree of genius would be needed to discover the exact nature of these relations. Such an hypothesis naturally suggests itself. But it must be admitted that it does not seem sufficient to account for the extraordinary accuracy with which these conceptions apply to the phenomena of Nature, and it is probable that there is some secret here that remains to be discovered.

IS THE UNIVERSE FINITE?

A boundless universe can have no design as a whole, and mystical theories face a strong presumption.

5.34 Some important questions of logic depend on whether we are to consider the material universe as limited in extent and finite in age, or as quite boundless in space and time. In the first case, it is conceivable that a general plan or design embracing the whole universe might be discovered, and it would be proper to be on the lookout for traces of such a unity. In the second case, since the part of the world we can have any experience of is less than the smallest assignable fraction, it follows that we could never discover any *pattern* in the universe except a repeating one. Any design embracing the whole would be beyond our

powers to discern, and beyond the combined powers of all intellects for all time. Now, what is absolutely incapable of being known is, as we saw in an earlier paper, not real at all. An absolutely unknowable existence is a nonsensical phrase. If, therefore, the universe is infinite, the attempt to find in it any design embracing it as a whole is futile, and involves a false way of looking at the subject. If the universe never had a beginning, and if in space world stretches beyond world without limit, then there is no *whole* of material things, and consequently no general character to the universe, and no need or possibility of any governor for it. But if there was a time before which absolutely no matter existed, if there are certain absolute bounds to the region of things, outside of which there is mere void, then we naturally look for an explanation of this; and since we cannot look for it among material things, the hypothesis of a great disembodied animal, the creator and governor of the world, is natural enough.

5.35 The actual state of the evidence about the limits of the universe is as follows. As to time, we find on our earth a constant progress of development since the planet was a red-hot ball; the solar system seems to have resulted from the condensation of a nebula, and the process appears to be still going on. We sometimes see stars (presumably with systems of worlds) destroyed and apparently resolved back into the nebulous condition, but we have no evidence of any existence of the world before the nebulous stage from which it seems to have evolved. All this, on the whole, rather favors the idea of a beginning. As for limits in space, we cannot be sure that we see anything outside the system of the Milky Way. Minds with theological leanings therefore have no need to distort the facts to reconcile them with their views.

5.36 But the only scientific presumption is that the unknown parts of space and time are, like the known parts, occupied. Since we see cycles of life and death in all development that we can trace to the end, the presumption is that the same holds for solar systems. And since enormous distances lie between the

different planets of our solar system relative to their diameters, and still more enormous distances lie between our system and other systems relative to its diameter, it may be supposed that other galactic clusters exist so remote from ours that they cannot be recognized as such with certainty. I do not say that these are strong inductions. I only say that they are the presumptions which, in our ignorance of the facts, should be preferred to hypotheses involving conceptions of things and occurrences totally different in character from anything we have ever experienced—such as disembodied spirits, the creation of matter, violations of the laws of mechanics, and so on.

5.37 The universe ought to be presumed too vast to have any character. When it is claimed that the arrangements of Nature are benevolent, or just, or wise, or of any other particular kind, we ought to be prejudiced against such opinions, as the offspring of an ill-founded notion that the world is finite. And examination has so far shown that such benevolence, justice, and so on are of a most limited kind—limited in degree and limited in range.

5.38 In the same way, if anyone claims to have discovered a plan in the structure of living beings, or a scheme in their classification, or a regular arrangement among natural objects, or a system of proportion in the human form, or an order of development, or a correspondence between conjunctions of the planets and human events, or a significance in numbers, or a key to dreams, the first thing we must ask is whether such relations can be explained on mechanical principles. If not, they should be viewed with disfavor, as already having a strong presumption against them; and examination has generally exploded all such theories.

5.39 There are minds to whom every prejudice, every presumption, seems unfair. It is easy to say what minds these are. They belong to people who have never known what it is to draw a well-grounded induction, and who imagine that other people's knowledge is as nebulous as their own. That all science turns on presumption (not of a formal but of a real kind) is no argument

for them, because they cannot imagine that there is anything solid in human knowledge. These are the people who waste their time and money on perpetual-motion machines and other such rubbish.

5.40 But there are better minds who take up mystical theories (by which I mean all theories that cannot possibly be explained mechanically). These are people strongly prejudiced in favor of such theories. We all have natural tendencies to believe in such things; our education often strengthens this tendency; and the result is that to many minds nothing seems so probable in advance as a theory of this kind. Such people find evidence enough in favor of their views, and in the absence of any recognized logic of induction they cannot be driven from their belief.

5.41 But to the mind of a physicist there ought to be a strong presumption against every mystical theory. And so it seems to me that those scientific men who have tried to make out that science is not hostile to theology have not been as clear-sighted as their opponents.

5.42 It would be extravagant to say that science can at present disprove religion; but it does seem to me that the spirit of science is hostile to any religion except one like that of M. Vacherot. Our appointed teachers tell us that Buddhism is a miserable and atheistic faith, stripped of the most glorious and necessary attributes of a religion; that its priests can be of no use to agriculture by praying for rain, nor to war by commanding the sun to stand still. We also hear the protests of those who warn us that to shake the general belief in the living God would be to shake general morals, public and private.[12] This, too, must be admitted. Such a revolution of thought could no more be accomplished without waste and desolation than a plantation of trees could be moved to new ground, however healthy that ground might be, without all of them languishing for a time and many of them dying. Nor, by the way, should it be presumed that a man would have taken part in a movement with a possibly atheistic outcome without having seriously and adequately con-

12 The sun standing still alludes to Joshua 10:12–13.

sidered that responsibility. But let the consequences of such a belief be as dire as they may, one thing is certain: the state of the facts, whatever it is, will surely be found out, and no human prudence can long halt the triumphal chariot of truth—no, not even if the discovery were such as to drive every member of our race to suicide!

5.43 But it would be folly to suppose that any metaphysical theory about the mode of being of the perfect is going to destroy that aspiration toward the perfect which constitutes the essence of religion. It is true that if the priests of some particular form of religion succeed in making it generally believed that religion cannot exist without accepting certain formulas, or if they succeed in weaving certain dogmas so tightly into popular religion that people can see no essential likeness between a religion that accepts these points of faith and one that rejects them, the result may very well be to make irreligious those who cannot believe these things. Nor can we ever hope that any body of priests will regard themselves as teachers of religion in general more than of the particular system of theology their own party advocates. But no one need be shut out from sharing in the common feelings, or from as much of their public expression as is open to all the laity, by the unphilosophical narrowness of those who guard the mysteries of worship. Am I to be kept from joining in that common joy at the revelation of enlightened principles of religion which we celebrate at Easter and Christmas, because I think that certain scientific, logical, and metaphysical ideas that have been mixed up with these principles are untenable? No; to do so would be to rate those errors as more important than the truth—an opinion few would accept. People who do not believe what are really the fundamental principles of Christianity are rare, and all but these few ought to feel at home in the churches.

Deduction, Induction, and Hypothesis

The three kinds of inference, their rules illustrated with bags of beans, and what each can and cannot establish.

DEDUCTION, INDUCTION, AND HYPOTHESIS I.

6.1 The chief business of the logician is to classify arguments, for all testing clearly depends on classification. The logicians' classes are defined by certain typical forms called syllogisms. For example, the syllogism called *Barbara* goes as follows:[1]

6.2 S is M; M is P: / Hence, S is P.

6.3 Or, putting words in place of letters—

6.4 Enoch and Elijah were men; all men die: / Hence, Enoch and Elijah must have died.[2]

6.5 The logicians' "is P" stands for any verb, active or intransitive. It can be strictly proved (though I will not trouble the reader with the proof) that all arguments whatever can be put into this form—but only on the condition that the "is" means "*is* for the purposes of the argument" or "is represented by." Thus an induction will look something like this in this form:

6.6 These beans are two-thirds white;
But, the beans in this bag are (represented by) these beans;
∴The beans in the bag are two-thirds white.

6.7 But the fact that all inference can in some way be reduced to *Barbara* does not mean that this is the most appropriate form

1 *Barbara* is the medieval mnemonic name for the syllogism whose three propositions are all universal affirmatives (the three a's): all M is P; all S is M; therefore all S is P.

in which to represent every kind of inference. On the contrary, to show the distinctive characters of different sorts of inference, they clearly have to be displayed in different forms peculiar to each. *Barbara* is the particular type of deductive reasoning; and as long as the "is" is taken literally, no inductive reasoning can be put into this form. *Barbara* is, in fact, nothing but the application of a rule. The so-called major premise lays down this rule: for example, *All men are mortal.* The other, or minor, premise states a case under the rule: *Enoch was a man.* The conclusion applies the rule to the case and states the result: *Enoch is mortal.* All deduction is of this character; it is merely the application of general rules to particular cases. Sometimes this is not very evident, as in the following:

6.8 All quadrangles are figures,
But no triangle is a quadrangle;
Therefore, some figures are not triangles.

6.9 But here the reasoning is really this:

6.10 *Rule*.—Every quadrangle is other than a triangle.
Case.—Some figures are quadrangles.
Result.—Some figures are not triangles.

6.11 Inductive or synthetic reasoning, being something more than the mere application of a general rule to a particular case, can never be reduced to this form.

6.12 If we take one bean at random from a bag of beans of which we know that 2/3 are white, it is a deductive inference that this bean is probably white, the probability being 2/3. In effect, we have the following syllogism:

6.13 *Rule*.—The beans in this bag are 2/3 white.
Case.—This bean has been drawn in such a way that in the long run the relative number of white beans so drawn would be equal to the relative number in the bag.
Result.—This bean has been drawn in such a way that in the long run it would turn out white 2/3 of the time.

2 Enoch and Elijah: biblical figures said to have been taken up to heaven without dying (Genesis 5:24; 2 Kings 2:11).

6.14 If instead of drawing one bean we draw a handful at random and conclude that about 2/3 of the handful are probably white, the reasoning is of the same sort. If, however, not knowing what proportion of the beans in the bag are white, we draw a handful at random and, finding 2/3 of the beans in the handful white, conclude that about 2/3 of those in the bag are white, then we are rowing against the current of deductive sequence, and concluding a rule from the observation of a result in a certain case. This is especially clear when the whole handful turns out to be one color. The induction then is:

6.15 These beans were in this bag.
These beans are white.
∴ All the beans in the bag were white.
This is merely an inversion of the deductive syllogism:
Rule. All the beans in the bag were white.
Case. These beans were in the bag
Result. These beans are white.

6.16 So induction is the inference of the *rule* from the *case* and *result*.

MAKING AN HYPOTHESIS

Inferring a case from a rule and a result is a third kind of inference, shown with beans.

6.17 But this is not the only way of inverting a deductive syllogism so as to produce a synthetic inference. Suppose I enter a room and find there a number of bags containing different kinds of beans. On the table there is a handful of white beans, and after some searching I find that one of the bags contains only white beans. I at once infer, as a probability or as a fair guess, that this handful was taken out of that bag. This sort of inference is called *making an hypothesis*. It is the inference of a case from a rule and result. We have, then—

6.18 Deduction.
Rule.—All the beans from this bag are white.
Case.—These beans are from this bag.
∴ *Result*.—These beans are white.
Induction.
Case.—These beans are from this bag.
Result.—These beans are white.
∴ *Rule*.—All the beans from this bag are white.
Hypothesis.
Rule.—All the beans from this bag are white.
Result.—These beans are white.
∴ *Case*.—These beans are from this bag.

6.19 We therefore classify all inference as follows:

6.20 Inference.
Deductive or Analytic. Synthetic.
Induction. Hypothesis.

6.21 Induction is where we generalize from a number of cases of which something is true, and infer that the same thing is true of a whole class. Or it is where we find a certain thing to be true of a certain proportion of cases, and infer that it is true of the same proportion of the whole class. Hypothesis is where we find some very curious circumstance, which would be explained by the supposition that it was a case of a certain general rule, and thereupon adopt that supposition. Or it is where we find that in certain respects two objects strongly resemble each other, and infer that they strongly resemble each other in other respects.

6.22 I once landed at a seaport in a Turkish province. As I was walking up to the house I was to visit, I met a man on horseback, surrounded by four horsemen holding a canopy over his head. Since the governor of the province was the only person I could think of who would be so greatly honored, I inferred that this was he. This was an hypothesis.

6.23 Fossils are found—say, remains like those of fishes, but far in the interior of the country. To explain the phenomenon, we

suppose that the sea once washed over this land. This is another hypothesis.

6.24 Countless documents and monuments refer to a conqueror called Napoleon Bonaparte. Though we have not seen the man, we cannot explain what we have seen—namely, all these documents and monuments—without supposing that he really existed. Hypothesis again.

6.25 As a general rule, hypothesis is a weak kind of argument. It often inclines our judgment so slightly toward its conclusion that we cannot say we believe the conclusion to be true; we only surmise that it may be so. But there is no difference, except one of degree, between such an inference and the one by which we are led, from our feeling as if we remembered the occurrences of yesterday, to believe that we do.

THE INDIRECT MOODS

Baroco and Bocardo, made probable, correspond to hypothesis and induction.

6.26 Besides the way just pointed out of inverting a deductive syllogism to produce an induction or hypothesis, there is another. If the truth of a certain conclusion would necessarily follow from the truth of a certain premise, then the falsity of the premise would follow from the falsity of the conclusion. Take, for instance, the following syllogism in *Barbara*:

6.27 *Rule*.—All men are mortal.
Case.—Enoch and Elijah were men.
∴ *Result*.—Enoch and Elijah were mortal.

6.28 Now, a person who denies this result may admit the rule, and in that case he must deny the case. Thus:

6.29 *Denial of Result*.—Enoch and Elijah were not mortal.
Rule.—All men are mortal.
∴ *Denial of Case*.—Enoch and Elijah were not men.

6.30 This kind of syllogism is called *Baroco*, which is the typical mood of the second figure. On the other hand, the person who

denies the result may admit the case, and in that case he must deny the rule. Thus:[3]

6.31 *Denial of the Result*.—Enoch and Elijah were not mortal.
Case.—Enoch and Elijah were men.
∴ *Denial of the Rule*.—Some men are not mortal.

6.32 This kind of syllogism is called *Bocardo*, which is the typical mood of the third figure.

6.33 *Baroco* and *Bocardo* are, of course, deductive syllogisms, but of a very peculiar kind. Logicians call them indirect moods, because they need some transformation before they appear as the application of a rule to a particular case. But if, instead of starting as we have done here with a necessary deduction in *Barbara*, we take a probable deduction of similar form, the indirect moods we obtain will be—

6.34 Corresponding to *Baroco*, an hypothesis; / and, Corresponding to *Bocardo*, an induction.

6.35 For example, let us begin with this probable deduction in *Barbara*:

6.36 *Rule*.—Most of the beans in this bag are white.
Case.—This handful of beans is from this bag.
∴ *Result*.—Probably, most of this handful of beans are white.

6.37 Now, deny the result, but accept the rule:

6.38 *Denial of Result*.—Few beans of this handful are white.
Rule.—Most beans in this bag are white.
∴ *Denial of Case*.—Probably, these beans were taken from another bag.

6.39 This is an hypothetical inference. Next, deny the result, but accept the case:

6.40 *Denial of Result*.—Few beans of this handful are white.
Case.—These beans came from this bag.
∴ *Denial of Rule*.—Probably, few beans in the bag are white.

3 In the old logic a "figure" is determined by the position of the middle term in the premises, and a "mood" by the kinds of propositions (universal or particular, affirmative or negative) that make up the syllogism; *Baroco* is the mnemonic name of one such mood.

6.41 This is an induction.

6.42 The relation thus shown between synthetic and deductive reasoning is not without its importance. When we adopt a certain hypothesis, it is not only because it will explain the observed facts, but also because the contrary hypothesis would probably lead to results contrary to those observed. Likewise, when we make an induction, it is drawn not only because it explains the distribution of characters in the sample, but also because a different rule would probably have led to the sample being other than it is.

6.43 But the advantage of this way of looking at the subject could easily be overrated. An induction is really the inference of a rule, and to consider it as the denial of a rule is an artificial conception. It is admissible only because, when statistical or proportional propositions are considered as rules, the denial of a rule is itself a rule. Likewise, an hypothesis is really a subsumption of a case under a class and not the denial of one—except in this sense, that to deny a subsumption under one class is to admit a subsumption under another.

6.44 *Bocardo* may be considered as an induction so timid that it loses its amplifiative character entirely. Enoch and Elijah are specimens of a certain kind of men. These instances show that all that kind of men are immortal. But instead of boldly concluding that all very pious men, or all men who are favorites of the Almighty, etc., are immortal, we refrain from specifying the description of men, and rest content with the merely explicative inference that some men are immortal. In the same way, *Baroco* might be considered as a very timid hypothesis. Enoch and Elijah are not mortal. Now, we might boldly suppose them to be gods or something of that sort; but instead we limit ourselves to the inference that they are of some nature different from that of man.

6.45 But, after all, there is an immense difference between the relation of *Baroco* and *Bocardo* to *Barbara* and that of Induction and Hypothesis to Deduction. *Baroco* and *Bocardo* are based on the fact that if the truth of a conclusion necessarily follows from

the truth of a premise, then the falsity of the premise follows from the falsity of the conclusion. This is always true. It is different when the inference is only probable. It by no means follows that, because the truth of a certain premise would make the truth of a conclusion probable, the falsity of the conclusion therefore makes the falsity of the premise probable. At least, as we saw in an earlier paper, this is true only when the word "probable" is used in one sense in the antecedent and in another in the consequent.

RULES FOR HYPOTHESIS

The torn paper shows how hypothesis differs from induction, and three rules make it trustworthy.

6.46 A certain anonymous piece of writing is on a torn piece of paper. A certain person is suspected of being the author. His desk, to which only he has had access, is searched, and in it is found a piece of paper whose torn edge exactly fits, in all its irregularities, the edge of the paper in question. It is a fair hypothetic inference that the suspected man was actually the author. The ground of this inference is evidently that two torn pieces of paper are extremely unlikely to fit together by accident. Therefore, of a great number of inferences of this sort, only a very small proportion would be deceptive. The analogy of hypothesis with induction is so strong that some logicians have confused the two. Hypothesis has been called an induction of characters. A number of characters belonging to a certain class are found in a certain object, and from this it is inferred that all the characters of that class belong to the object in question. This certainly involves the same principle as induction, though in a modified form. In the first place, characters cannot simply be enumerated, as objects can; in the second place, characters run in categories. When we make an hypothesis like the one about the piece of paper, we examine only a single line of characters, or perhaps two or three, and we take no specimen at all of others. If hypothesis were nothing but an induction, all we should be jus-

tified in concluding in the example above would be that the two pieces of paper, which matched in the irregularities examined, would be found to match in other—say, slighter—irregularities. The inference from the shape of the paper to its ownership is precisely what distinguishes hypothesis from induction, and makes it a bolder and more perilous step.

6.47 The same warnings that have been given against imagining that induction rests on the uniformity of Nature might be repeated with regard to hypothesis. Here, as there, such a theory not only utterly fails to account for the validity of the inference, but also gives rise to methods of conducting it which are absolutely vicious. There are, no doubt, certain uniformities in Nature, the knowledge of which will greatly fortify an hypothesis. For example, we suppose that iron, titanium, and other metals exist in the sun, because we find in the solar spectrum many lines that coincide in position with those these metals would produce; and this hypothesis is greatly strengthened by our knowledge of how remarkably distinctive the particular line of characters observed is. But such a fortification of hypothesis is of a deductive kind, and an hypothesis may still be probable when such reinforcement is lacking.

6.48 There is no greater or more frequent mistake in practical logic than to suppose that things which strongly resemble one another in some respects are any the more likely, for that reason, to be alike in others. That this is absolutely false can be rigorously demonstrated. But since the reasoning is somewhat severe and complicated (requiring, like all such reasoning, the use of A, B, C, etc., to set it out), the reader would probably find it distasteful, and I omit it. An example, however, may illustrate the proposition. The comparative mythologists busy themselves with finding points of resemblance between solar phenomena and the careers of the heroes of all sorts of traditional stories, and on the basis of such resemblances they infer that these heroes are personifications of the sun. If there is anything more to their reasoning, it has never been made clear to me. An ingenious logician, to show how futile all this is, wrote a little book in

which he pretended to prove, in the same manner, that Napoleon Bonaparte is only a personification of the sun. It was really wonderful to see how many points of resemblance he made out.[4] The truth is that any two things resemble each other just as strongly as any two others, if obscure resemblances are admitted. But in order for the process of making an hypothesis to lead to a probable result, the following rules must be followed:

6.49 1. The hypothesis should be clearly put as a question before making the observations that are to test its truth. In other words, we must try to see what the result of predictions from the hypothesis will be.

6.50 2. The respect in which the resemblances are noted must be taken at random. We must not take a particular kind of prediction for which the hypothesis is known to be good.

6.51 3. The failures as well as the successes of the predictions must be honestly noted. The whole proceeding must be fair and unbiased.

6.52 Some people imagine that bias and counter-bias help to extract the truth—that hot, partisan debate is the way to investigate. This is the theory behind our atrocious legal procedure. But Logic puts its heel on this suggestion. It irrefutably demonstrates that knowledge can be furthered only by the real desire for it, and that the methods of obstinacy and of authority, and every way of trying to reach a foregone conclusion, are of absolutely no value. These things are proved. The reader is free to think so or not, as long as the proof is not set out, or as long as he refrains from examining it. In just the same way, he can keep his freedom of opinion about the propositions of geometry, if he likes; only, in that case, if he takes a fancy to read Euclid, he will do well to skip whatever he finds with A, B, C, etc., for if he reads that disagreeable matter attentively, his freedom of opinion about geometry may unhappily be lost forever.

4 The "solar mythology" school was associated above all with Max Müller. The satire is usually identified as Jean-Baptiste Pérès, *Comme quoi Napoléon n'a jamais existé* ("How Napoleon Never Existed," 1827).

6.53 How many people there are who are incapable of putting this question to their own consciences: "Do I want to know how the fact stands, or not?"

6.54 The rules laid down so far for induction and hypothesis are absolutely essential ones. There are many other maxims, expressing particular devices for making synthetic inferences strong, which are extremely valuable and should not be neglected—for example, Mr. Mill's four methods. Nevertheless, even when these are entirely neglected, inductions and hypotheses can, and sometimes do, attain the greatest force.[5]

INDUCTION AND HYPOTHESIS IN PHYSICS

Empirical formulas, laws of Nature and the kinetic theory of gases show the two kinds working together.

6.55 Classifications that are perfectly satisfactory in all cases hardly exist. Even with regard to the great distinction between explicative and ampliative inferences, examples could be found that seem to lie on the border between the two classes and to share in some respects the characters of both. The same is true of the distinction between induction and hypothesis. On the whole, it is broad and decided. By induction, we conclude that facts similar to observed facts are true in cases not examined. By hypothesis, we conclude the existence of a fact quite different from anything observed, from which, according to known laws, something observed would necessarily result. The former is reasoning from particulars to the general law; the latter, from effect to cause. The former classifies; the latter explains. Only in some special cases can there be more than a momentary doubt as to which category a given inference belongs to. One exception is where we observe, not similar facts under similar circumstances, but different facts under different circumstances—the difference of the former, however, bearing a definite relation to the difference of the latter. Such inferences, which are really

5 Mill's four methods: the methods of agreement, difference, residues, and concomitant variations.

inductions, nevertheless sometimes show some unmistakable resemblances to hypotheses.

6.56 Knowing that water expands with heat, we make a number of observations of the volume of a constant mass of water at different temperatures. Scrutinizing a few of these suggests a form of algebraic formula that will approximately express the relation of the volume to the temperature. It may be, for instance, that, v being the relative volume and t the temperature, the few observations examined indicate a relation of the form—

6.57 $v = 1 + at + bt^2 + ct^3$.

6.58 On examining observations at other temperatures, taken at random, this idea is confirmed; and we draw the inductive conclusion that all observations within the limits of temperature from which we have drawn our observations could equally be satisfied in this way. Once we have established that such a formula is possible, it is a mere matter of arithmetic to find the values of a, b, and c that will make the formula best satisfy the observations. This is what physicists call an *empirical formula*, because it rests on mere induction and is not explained by any hypothesis.

6.59 Such formulas, though very useful as a means of describing the results of observations in general terms, do not rank high among scientific discoveries. The induction they embody—that expansion by heat (or whatever other phenomenon is in question) takes place in a perfectly gradual manner, without sudden leaps or innumerable fluctuations—although really important, attracts no attention, because it is what we naturally expect. But the defects of such expressions are very serious. In the first place, as long as the observations are subject to error, as all observations are, the formula cannot be expected to satisfy them exactly. But the discrepancies cannot be due solely to the errors of the observations; they must be partly owing to the error of the formula, which has been deduced from erroneous observations. Moreover, we have no right to suppose that the real facts, if they could be had free from error, could be expressed by such a formula at all. They might perhaps be

expressed by a similar formula with an infinite number of terms; but what use would that be to us, since it would require an infinite number of coefficients to be written down? When one quantity varies with another, and the corresponding values are exactly known, it is a mere matter of mathematical ingenuity to find some way of expressing their relation simply. If one quantity is of one kind—say, a specific gravity—and the other of another kind—say, a temperature—we do not wish to find an expression for their relation that is wholly free from numerical constants. For if it were free from them when, say, specific gravity compared with water and temperature on the centigrade thermometer were in question, numbers would have to be introduced when the scales of measurement were changed. We may, however, and do, wish to find formulas expressing the relations of physical phenomena that contain no more arbitrary numbers than changes in the scales of measurement might require.

6.60 When a formula of this kind is discovered, it is no longer called an empirical formula but a law of Nature, and sooner or later it is made the basis of an hypothesis meant to explain it. These simple formulas are not usually, if ever, exactly true, but they are none the less important for that; and the great triumph of the hypothesis comes when it explains not only the formula but also the deviations from it. In the current language of physicists, an hypothesis of this importance is called a theory, while the term hypothesis is restricted to suggestions that have little evidence in their favor. There is some justice in the contempt that clings to the word hypothesis. To think that we can strike out of our own minds a true preconception of how Nature acts is a vain fancy. As Lord Bacon rightly says: "The subtlety of Nature far exceeds the subtlety of sense and intellect: so that these fine meditations, and speculations, and reasonings of men are a sort of insanity, only there is no one at hand to remark it." The successful theories are not pure guesses, but are guided by reasons.[6]

6.61 The kinetic theory of gases is a good example of this. The theory is meant to explain certain simple formulas, the chief of which is called Boyle's law. It says that if air or any other gas is placed in a cylinder with a piston, and its volume is measured under the pressure of the atmosphere—say, fifteen pounds per square inch—and if another fifteen pounds per square inch is then placed on the piston, the gas will be compressed to half its bulk, and in a similar inverse ratio for other pressures.[7] The hypothesis that has been adopted to account for this law is that the molecules of a gas are small, solid particles at great distances from each other (relative to their size), moving with great velocity, without noticeable attractions or repulsions, until they happen to come very close to one another. Grant this, and it follows that when a gas is under pressure, what keeps it from collapsing is not the incompressibility of the separate molecules—which are under no pressure at all, since they do not touch—but the pounding of the molecules against the piston. The further the piston falls and the more the gas is compressed, the closer together the molecules will be; the greater the number of them at any moment within a given distance of the piston; the shorter the distance any one of them will travel before its course is changed by the influence of another; the greater the number of new courses each takes in a given time; and the more often each one within a given distance of the piston will strike it. This explains Boyle's law. The law is not exact; but then the hypothesis does not lead us to it exactly. For, in the first place, if the molecules are large, they will strike each other more often when their mean distances are reduced, and will consequently strike the piston more often and produce more pressure on it. On the other hand, if the molecules attract one another, they will remain for a noticeable time within one another's influence; consequently they will not strike the wall as often as they other-

6 From the *Novum Organum*, Book I, aphorism 10.

7 Boyle's law, named for Robert Boyle (1627–1691): at constant temperature, the volume of a gas varies inversely with its pressure.

wise would, and the pressure will be increased less by compression.

6.62 When the kinetic theory of gases was first proposed by Daniel Bernoulli in 1738, it rested only on Boyle's law, and was therefore pure hypothesis. It was accordingly quite naturally and deservedly neglected. But at present the theory looks quite different. Not to speak of the considerable number of observed facts of different kinds with which it has been brought into relation, it is supported by the mechanical theory of heat. That bringing together bodies that attract one another, or separating bodies that repel one another, when sensible motion is neither produced nor destroyed, is always accompanied by the evolution of heat, is little more than an induction.[8] Now, it has been shown by experiment that when a gas is allowed to expand without doing work, a very small amount of heat disappears. This proves that the particles of the gas attract one another slightly—but only very slightly. It follows that when a gas is under pressure, what keeps it from collapsing is not any repulsion between the particles, since there is none. Now, there are only two modes of force known to us: force of position, or attractions and repulsions, and force of motion. Since, therefore, it is not force of position that gives a gas its expansive force, it must be force of motion. Seen from this point of view, the kinetic theory of gases appears as a deduction from the mechanical theory of heat. It should be observed, however, that it supposes that the same law of mechanics (that there are only those two modes of force) which holds for bodies such as we can see and examine also holds for things that are very different, the molecules of bodies. Such a supposition has only slender support from induction. Our belief in it is greatly strengthened by its connection with Boyle's law, and it is therefore to be considered an hypothetical inference. Yet it must be admitted that the kinetic theory of gases would deserve little credence if it had not been connected with the principles of mechanics.

6.63 The great difference between induction and hypothesis is that the former infers the existence of phenomena such as we have observed in cases that are similar, while hypothesis supposes something of a different kind from what we have directly observed, and frequently something it would be impossible for us to observe directly. Accordingly, when we stretch an induction well beyond the limits of our observation, the inference takes on the nature of hypothesis. It would be absurd to say that we have no inductive warrant for a generalization extending a little beyond the limits of experience, and there is no line to be drawn beyond which we cannot push our inference; only, it becomes weaker the further it is pushed. Yet if an induction is pushed very far, we cannot give it much credence unless we find that such an extension explains some fact that we can and do observe. Here, then, we have a kind of mixture of induction and hypothesis supporting one another; and most of the theories of physics are of this kind.

WHY THE DISTINCTION MATTERS

Induction is stronger, cannot replace hypothesis, forms habits, and helps classify the sciences.

6.64 That synthetic inferences may be divided into induction and hypothesis in the manner proposed here[9] is beyond question. The usefulness and value of the distinction are to be tested by its applications.

6.65 Induction is plainly a much stronger kind of inference than hypothesis, and this is the first reason for distinguishing between them. Hypotheses are sometimes regarded as provisional expedients, which in the progress of science are to be replaced by inductions. But this is a false view of the subject. Hypothetic reasoning very frequently infers a fact that cannot

8 Daniel Bernoulli (1700–1782), Swiss mathematician, in his *Hydrodynamica* (1738).
9 PEIRCE'S NOTE. This division was first made in a course of lectures by the author before the Lowell Institute, Boston, in 1866, and was printed in the "Proceedings of the American Academy of Arts and Sciences" for April 9, 1867.

be directly observed. It is an hypothesis that Napoleon Bonaparte once existed. How is that hypothesis ever to be replaced by an induction? It may be said that from the premise that such facts as we have observed are as they would be if Napoleon existed, we are to infer by induction that all facts still to be observed will be of the same character. There is no doubt that every hypothetic inference can be distorted in this way into the appearance of an induction. But the essence of an induction is that it infers from one set of facts another set of similar facts, whereas hypothesis infers from facts of one kind to facts of another. Now, the facts that serve as grounds for our belief in the historical reality of Napoleon are by no means necessarily the only kind of facts explained by his existence. It may be that, at the time of his career, events were being recorded in some way not now dreamed of: that some ingenious creature on a neighboring planet was photographing the earth, and that these pictures, on a sufficiently large scale, may some day come into our possession; or that some mirror on a distant star will, when the light reaches it, reflect the whole story back to earth. Never mind how improbable these suppositions are; everything that happens is infinitely improbable. I am not saying that *these* things are likely to occur, but that *some* effect of Napoleon's existence which now seems impossible is nevertheless certain to be brought about. The hypothesis asserts that such facts, when they do occur, will be of a kind to confirm, and not to refute, the existence of the man. In the impossibility of inferring hypothetical conclusions inductively, we have a second reason for distinguishing between the two kinds of inference.

6.66 A third merit of the distinction is that it is associated with an important psychological, or rather physiological, difference in the way facts are apprehended. Induction infers a rule. Now, the belief in a rule is a habit. That a habit is a rule active in us is evident. That every belief is of the nature of a habit, insofar as it is of a general character, has been shown in the earlier papers of this series. Induction, therefore, is the logical formula that expresses the physiological process of forming a habit. Hypo-

thesis substitutes a single conception for a complicated tangle of predicates attached to one subject. Now, there is a peculiar sensation belonging to the act of thinking that each of these predicates inheres in the subject. In hypothetic inference, the complicated feeling so produced is replaced by a single feeling of greater intensity, the one belonging to the act of thinking the hypothetic conclusion. Now, when our nervous system is excited in a complicated way, and there is a relation between the elements of the excitation, the result is a single harmonious disturbance, which I call an emotion. Thus the various sounds made by the instruments of an orchestra strike the ear, and the result is a peculiar musical emotion, quite distinct from the sounds themselves. This emotion is essentially the same thing as an hypothetic inference, and every hypothetic inference involves the formation of such an emotion. We may say, therefore, that hypothesis produces the *sensuous* element of thought, and induction the *habitual* element. As for deduction, which adds nothing to the premises but only selects one of the various facts represented in them and brings the attention down to it, this may be considered the logical formula for paying attention, which is the *volitional* element of thought and corresponds to nervous discharge in the sphere of physiology.

6.67 Another merit of the distinction between induction and hypothesis is that it leads to a very natural classification of the sciences and of the minds that pursue them. What must separate different kinds of scientific men more than anything else are the differences in their *techniques*. We cannot expect men who work chiefly with books to have much in common with men whose lives are spent in laboratories. But after differences of this kind, the next most important are differences in the modes of reasoning. Among the natural sciences we have, first, the classificatory sciences, which are purely inductive—systematic botany and zoology, mineralogy, and chemistry. Then we have the sciences of theory, as explained above—astronomy, pure physics, etc. Then we have sciences of hypothesis—geology, biology, etc.

6.68 There are many other advantages of the distinction in question, which I shall leave the reader to discover by experience. If he will only make a habit of considering whether a given inference belongs to one or the other of the two forms of synthetic inference given on page 472, I can promise him that he will profit by it in various ways.[10]

10 Page 472 of the original magazine printing: the schemata of Deduction, Induction, and Hypothesis given above (6.18).