Tractatus Logico-Philosophicus

LUDWIG WITTGENSTEIN
1889–1951

Tractatus Logico-Philosophicus

Logisch-philosophische Abhandlung

Translated from the German with AI (Claude) and edited

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Tractatus Logico-Philosophicus · Ludwig Wittgenstein (1889–1951)

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INTRODUCTION

Ludwig Wittgenstein wrote the *Tractatus* as a soldier in the First World War and published it in 1921. It is short, around 20,000 words, and built as a tree of numbered propositions: seven main ones, each explained by the decimal propositions beneath it.

It sets out to show how language pictures the world, and so where the limits of what can be said lie. Logic, ethics, the self and the meaning of life all turn out to lie beyond those limits: they can be shown, but not said. It ends with one of the most famous sentences in philosophy.

Wittgenstein thought the book had solved the problems of philosophy. Later he rejected much of it.

Preface

Vorwort
Wittgenstein says the book draws a limit to what can be said and claims to have solved philosophy's problems.

Motto ...and everything that one knows, and has not merely heard as rumbling and roaring, can be said in three words. Kürnberger[1]

Vorwort Perhaps this book will be understood only by someone who has already had the thoughts expressed in it—or at least similar thoughts—himself.—So it is not a textbook.—Its purpose would be achieved if it gave pleasure to one person who read it with understanding.

The book deals with the problems of philosophy and shows—as I believe—that the way these problems are posed rests on a misunderstanding of the logic of our language. The whole sense of the book could be summed up roughly in these words: What can be said at all can be said clearly; and what we cannot talk about we must pass over in silence.

So the book aims to draw a limit to thinking, or rather—not to thinking, but to the expression of thoughts. For in order to draw a limit to thinking, we would have to be able to think both sides of this limit (we would, that is, have to be able to think what cannot be thought).

So the limit can be drawn only in language, and what lies beyond the limit will simply be nonsense.

How far my efforts coincide with those of other philosophers I do not wish to judge. Indeed, what I have written here makes

no claim whatever to novelty in its details; and that is also why I cite no sources, since it is a matter of indifference to me whether what I have thought has already been thought by someone else before me.

I will mention only this: that I owe a large part of the stimulus to my thoughts to the magnificent works of Frege and to the writings of my friend Mr. Bertrand Russell.[2]

If this work has any value, it consists in two things. First, in the fact that thoughts are expressed in it, and this value will be the greater, the better the thoughts are expressed. The more squarely the nail has been hit on the head.—Here I am aware of having fallen far short of what is possible. Simply because my powers are too slight to master the task.—May others come and do it better.

On the other hand, the *truth* of the thoughts communicated here seems to me unassailable and definitive. I am therefore of the opinion that I have, in all essentials, solved the problems once and for all. And if I am not mistaken in this, then the value of this work consists, secondly, in showing how little has been achieved when these problems have been solved.

L. W.

Vienna, 1918

1 Ferdinand Kürnberger (1821–1879), Austrian writer and essayist.

2 Gottlob Frege (1848–1925), German logician and founder of modern logic; Bertrand Russell (1872–1970), British philosopher, Wittgenstein's teacher at Cambridge.

Proposition 1

The world is everything that is the case

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1 The world is everything that is the case.

The decimal numbers assigned to the individual propositions indicate the logical weight of the propositions, the emphasis laid on them in my exposition. The propositions n.1, n.2, n.3, etc., are remarks on proposition no. n; the propositions n.m1, n.m2, etc., are remarks on proposition no. n.m; and so on.

1.1 The world is the totality of facts, not of things.

1.11 The world is determined by the facts, and by their being *all* the facts.

1.12 For the totality of facts determines what is the case, and also whatever is not the case.

1.13 The facts in logical space are the world.

1.2 The world divides into facts.

1.21 Any one of them can be the case or not be the case, and everything else remain the same.

Proposition 2

What is the case, the fact, is the obtaining of states of affairs

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2 What is the case, the fact, is the obtaining of states of affairs.

2.01 A state of affairs is a combination of objects (items, things).

2.011 It is essential to a thing that it can be a constituent of a state of affairs.

2.012 In logic nothing is accidental: if a thing *can* occur in a state of affairs, the possibility of that state of affairs must already be prejudged in the thing.

2.0121 It would seem, so to speak, a matter of chance if a situation turned out, after the fact, to fit a thing that could subsist entirely on its own.

If things can occur in states of affairs, this must already lie in them.

(Something logical cannot be merely possible. Logic deals with every possibility, and all possibilities are its facts.)

Just as we cannot think of spatial objects at all outside space, or of temporal objects outside time, so we cannot think of *any* object outside the possibility of its combination with others.

If I can think of an object as combined in a state of affairs, I cannot think of it outside the *possibility* of this combination.

2.0122 A thing is independent insofar as it can occur in all *possible* situations, but this form of independence is a form of connection with states of affairs, a form of dependence. (It is impossible for words to appear in two different ways: on their own and in the proposition.)

2.0123 If I know an object, I also know all the possibilities of its occurring in states of affairs.

(Every such possibility must lie in the nature of the object.)

A new possibility cannot be discovered after the fact.

2.01231 To know an object, I need not know its external properties—but I must know all its internal properties.

2.0124 If all objects are given, then all *possible* states of affairs are thereby given as well.

2.013 Each thing is, as it were, in a space of possible states of affairs. I can think of this space as empty, but not of the thing without the space.

2.0131 A spatial object must lie in infinite space. (A point in space is an argument place.)

A spot in the visual field need not be red, but it must have some color: it has, so to speak, color space around it. A note must have *some* pitch, an object of touch *some* hardness, and so on.

2.014 Objects contain the possibility of all situations.

2.0141 The possibility of its occurring in states of affairs is the form of an object.

2.02 An object is simple.

2.0201 Every statement about complexes can be analyzed into a statement about their constituents and into those propositions that describe the complexes completely.

2.021 Objects make up the substance of the world. That is why they cannot be composite.

2.0211 If the world had no substance, then whether a proposition had sense would depend on whether another proposition was true.

2.0212 It would then be impossible to sketch a picture of the world (true or false).

2.022 It is obvious that even a world imagined as differing ever so much from the real one must have something—a form—in common with the real world.

2.023 This fixed form consists precisely of the objects.

2.0231 The substance of the world *can* determine only a form, and no material properties. For these are represented only by propositions—formed only by the configuration of objects.

2.0232 Incidentally: objects are colorless.

2.0233 Two objects of the same logical form are—apart from their external properties—distinguished from each other only by being different.

2.02331 Either a thing has properties that no other thing has, in which case one can simply single it out from the others by a description and point to it; or else there are several things that have all their properties in common, in which case it is quite impossible to point to any one of them.

For if nothing singles the thing out, I cannot single it out, since otherwise it would, after all, be singled out.

2.024 Substance is what subsists independently of what is the case.

2.025 It is form and content.

2.0251 Space, time and color (being colored) are forms of objects.

2.026 Only if there are objects can there be a fixed form of the world.

2.027 The fixed, the subsistent and the object are one.

2.0271 The object is the fixed, the subsistent; the configuration is the changing, the unstable.

2.0272 The configuration of objects forms the state of affairs.

2.03 In a state of affairs, objects hook into one another like the links of a chain.

2.031 In a state of affairs, objects stand to one another in a definite way.

2.032 The way in which objects hang together in a state of affairs is the structure of the state of affairs.

2.033 Form is the possibility of structure.

2.034 The structure of a fact consists of the structures of states of affairs.

2.04 The totality of obtaining states of affairs is the world.

2.05 The totality of obtaining states of affairs also determines which states of affairs do not obtain.

2.06 The obtaining and non-obtaining of states of affairs is reality.

(We also call the obtaining of states of affairs a positive fact, and their non-obtaining a negative fact.)

2.061 States of affairs are independent of one another.

2.062 From the obtaining or non-obtaining of one state of affairs, nothing can be inferred about the obtaining or non-obtaining of another.

2.063 The whole of reality is the world.

2.1 We picture facts to ourselves.[1]

2.11 A picture presents the situation in logical space, the obtaining and non-obtaining of states of affairs.

2.12 A picture is a model of reality.

2.13 In a picture, the elements of the picture correspond to the objects.

2.131 In a picture, the elements of the picture stand in for the objects.

2.14 A picture consists in its elements being related to one another in a definite way.

2.141 A picture is a fact.

2.15 That the elements of the picture are related to one another in a definite way presents the items as being related to one another in the same way.

Let this connection of the elements of the picture be called its structure, and the possibility of this structure its form of depiction.

2.151 The form of depiction is the possibility that things are related to one another in the same way as the elements of the picture.

2.1511 *That* is how a picture is linked to reality—it reaches right out to it.

2.1512 It is laid against reality like a ruler.

2.15121 Only the outermost points of the graduation marks *touch* the object being measured.

2.1513 On this view, then, the picture also includes the depicting relation, which makes it into a picture.

1 German *Bild* covers "picture", "image" and "model"; all three senses are in play here.

2.1514 The depicting relation consists of the correlations between the elements of the picture and the items.

2.1515 These correlations are, as it were, the feelers of the picture's elements, with which the picture touches reality.

2.16 In order to be a picture, a fact must have something in common with what it depicts.

2.161 In the picture and what it depicts, something must be identical, if the one is to be a picture of the other at all.

2.17 What a picture must have in common with reality in order to be able to depict it in its own way—correctly or incorrectly—is its form of depiction.

2.171 A picture can depict any reality whose form it has.

A spatial picture anything spatial, a colored one anything colored, etc.

2.172 A picture cannot, however, depict its form of depiction; it displays it.

2.173 A picture represents its subject from outside (its standpoint is its form of representation); that is why a picture represents its subject correctly or incorrectly.

2.174 But a picture cannot place itself outside its form of representation.

2.18 What every picture, of whatever form, must have in common with reality in order to be able to depict it at all—correctly or incorrectly—is logical form, that is, the form of reality.

2.181 If the form of depiction is logical form, the picture is called a logical picture.

2.182 Every picture is *also* a logical one. (On the other hand, not every picture is, for example, a spatial one.)

2.19 A logical picture can depict the world.

2.2 A picture has the logical form of depiction in common with what it depicts.

2.201 A picture depicts reality by representing a possibility of the obtaining and non-obtaining of states of affairs.

2.202 A picture represents a possible situation in logical space.

2.203 A picture contains the possibility of the situation that it represents.

2.21 A picture agrees with reality or it does not; it is correct or incorrect, true or false.

2.22 A picture represents what it represents, independently of its truth or falsity, through its form of depiction.

2.221 What a picture represents is its sense.

2.222 Its truth or falsity consists in the agreement or disagreement of its sense with reality.

2.223 In order to tell whether a picture is true or false, we must compare it with reality.

2.224 It is impossible to tell from the picture alone whether it is true or false.

2.225 There is no picture that is true a priori.

Proposition 3

The logical picture of the facts is the thought

3 The logical picture of the facts is the thought.

3.001 "A state of affairs is thinkable" means: we can picture it to ourselves.

3.01 The totality of true thoughts is a picture of the world.

3.02 The thought contains the possibility of the situation that it thinks. What is thinkable is also possible.

3.03 We cannot think anything illogical, because otherwise we would have to think illogically.

3.031 It used to be said that God could create everything except what would be contrary to the laws of logic.—For we could not *say* what an "illogical" world would look like.

3.032 It is as impossible to represent in language something that "contradicts logic" as it is, in geometry, to represent by its coordinates a figure that contradicts the laws of space, or to give the coordinates of a point that does not exist.

3.0321 We can indeed represent spatially a state of affairs that would contravene the laws of physics, but not one that would contravene the laws of geometry.

3.04 A thought that was correct a priori would be one whose possibility guaranteed its truth.

3.05 We could know a priori that a thought is true only if its truth could be recognized from the thought itself (without an object of comparison).

3.1 In the proposition, the thought is expressed in a way that can be perceived by the senses.

3.11 We use the perceptible sign of the proposition (spoken or written sign, etc.) as a projection of the possible situation.

The method of projection is the thinking of the proposition's sense.

3.12 The sign through which we express the thought I call the propositional sign. And the proposition is the propositional sign in its projective relation to the world.

3.13 Everything that belongs to the projection belongs to the proposition; but not what is projected.

So the possibility of what is projected, but not this itself.

Thus the proposition does not yet contain its sense, but it does contain the possibility of expressing it.

("The content of the proposition" means the content of the proposition that has sense.)

The proposition contains the form of its sense, but not its content.

3.14 The propositional sign consists in this: that its elements, the words, are related to one another within it in a definite way.

The propositional sign is a fact.

3.141 A proposition is not a jumble of words.—(Just as a musical theme is not a jumble of notes.)

A proposition is articulated.

3.142 Only facts can express a sense; a class of names cannot.

3.143 That the propositional sign is a fact is concealed by the ordinary form of expression in writing or print.

For in a printed proposition, for example, the propositional sign does not look essentially different from a word.

(This is how it was possible for Frege to call the proposition a compound name.)

3.1431 The essence of the propositional sign becomes very clear if we imagine it composed not of written signs but of spatial objects (such as tables, chairs, books).

The spatial arrangement of these things relative to one another then expresses the sense of the proposition.

3.1432 Not: "The complex sign 'aRb' says that a stands in the relation R to b", but rather: *That* "a" stands in a certain relation to "b" says *that* aRb.

3.144 Situations can be described, not *named*.

(Names are like points, propositions like arrows—they have sense.)

3.2 In a proposition the thought can be expressed in such a way that elements of the propositional sign correspond to the objects of the thought.

3.201 These elements I call "simple signs", and the proposition "completely analyzed".

3.202 The simple signs employed in propositions are called names.

3.203 A name means an object. The object is its meaning. ("A" is the same sign as "A".)[1]

3.21 The configuration of the simple signs in the propositional sign corresponds to the configuration of the objects in the situation.

3.22 In the proposition, the name stands in for the object.

3.221 Objects I can only *name*. Signs stand in for them. I can only speak *about* them; I cannot *put them into words*. A proposition can only say *how* a thing is, not *what* it is.

3.23 The requirement that simple signs be possible is the requirement that sense be determinate.

3.24 A proposition that is about a complex stands in an internal relation to a proposition about a constituent of that complex.

A complex can be given only by its description, and this description will either fit or not fit. A proposition in which a complex is mentioned will not be nonsensical if the complex does not exist, but simply false.

That an element of a proposition signifies a complex can be seen from an indeterminacy in the propositions in which it occurs. We *know* that not everything has yet been determined by

1 "Meaning" renders Frege's term *Bedeutung*: what a sign stands for (often translated "reference"), as distinct from its *Sinn*, "sense".

this proposition. (The notation for generality does, after all, *contain* a prototype.)

The contraction of the symbol of a complex into a simple symbol can be expressed by a definition.

3.25 There is one and only one complete analysis of the proposition.

3.251 A proposition expresses what it expresses in a definite, clearly specifiable way: the proposition is articulated.

3.26 A name cannot be dissected any further by a definition: it is a primitive sign.

3.261 Every defined sign signifies *via* the signs by which it was defined; and the definitions show the way.

Two signs, one a primitive sign and one defined by means of primitive signs, cannot signify in the same way. Names *cannot* be taken apart by definitions. (Nor can any sign that has a meaning alone, on its own.)

3.262 What does not come to expression in the signs, their application shows. What the signs swallow, their application states openly.

3.263 The meanings of primitive signs can be explained by elucidations. Elucidations are propositions that contain the primitive signs. So they can be understood only if the meanings of these signs are already known.

3.3 Only the proposition has sense; only in the context of a proposition does a name have meaning.

3.31 Every part of a proposition that characterizes its sense I call an expression (a symbol).

(The proposition itself is an expression.)

An expression is anything essential to the sense of a proposition that propositions can have in common with one another.

An expression marks out a form and a content.

3.311 An expression presupposes the forms of all the propositions in which it can occur. It is the common characteristic mark of a class of propositions.

3.312 It is therefore represented by the general form of the propositions that it characterizes.

And in this form the expression will be *constant* and everything else *variable*.

3.313 Thus an expression is represented by a variable whose values are the propositions that contain the expression.

(In the limiting case the variable becomes a constant, and the expression a proposition.)

I call such a variable a "propositional variable".

3.314 An expression has meaning only in a proposition. Every variable can be construed as a propositional variable.

(The variable name included.)

3.315 If we turn a constituent of a proposition into a variable, there is a class of propositions that are all values of the resulting variable proposition. In general this class still depends on what we mean, by arbitrary convention, by parts of the original proposition. But if we turn into variables all those signs whose meaning was arbitrarily fixed, there is still such a class. This class, however, now depends on no convention, but only on the nature of the proposition. It corresponds to a logical form—a logical prototype.

3.316 What values the propositional variable may take is stipulated.

The stipulation of the values *is* the variable.

3.317 The stipulation of the values of the propositional variable is the *specification of the propositions* whose common mark the variable is.

The stipulation is a description of these propositions.

So the stipulation will deal only with symbols, not with their meaning.

And *only* this is essential to the stipulation: *that it is merely a description of symbols and says nothing about what is signified.*

How the description of the propositions is carried out is inessential.

3.318 Like Frege and Russell, I construe a proposition as a function of the expressions contained in it.

3.32 The sign is the part of the symbol that can be perceived by the senses.

3.321 So two different symbols can have the sign (written sign or spoken sign, etc.) in common—they then signify in different ways.

3.322 The fact that we signify two objects with the same sign, but by two different *modes of signification*, can never indicate a characteristic the objects have in common. For the sign is, after all, arbitrary. So we could just as well choose two different signs, and then where would the common element in the signification be?

3.323 In everyday language it happens extremely often that the same word signifies in different ways—and so belongs to different symbols—or that two words that signify in different ways are used in the proposition in what outwardly looks the same way.

Thus the word "is" appears as the copula, as the sign of identity, and as the expression of existence; "exist" as an intransitive verb like "go"; "identical" as an adjective; we speak of *something*, but also of the fact that *something* happens.

(In the proposition "Green is green"—where the first word is a person's name and the last an adjective—these words do not simply have different meanings; they are *different symbols*.)

3.324 This is how the most fundamental confusions easily arise (the whole of philosophy is full of them).

3.325 To avoid these errors we must use a sign-language that excludes them, by not using the same sign in different symbols and by not using, in outwardly the same way, signs that signify in different ways. A sign-language, that is, that obeys *logical* grammar—logical syntax.

(The concept-script of Frege and Russell is such a language, although it does not yet exclude all mistakes.)[2]

3.326 To recognize the symbol in the sign, one must attend to its use with sense.

3.327 A sign determines a logical form only together with its logical-syntactic employment.

2 "Concept-script" translates *Begriffsschrift*, the name Frege gave to his logical notation (1879).

3.328 If a sign is *not used*, it is meaningless. That is the sense of Occam's maxim.

(If everything behaves as though a sign had meaning, then it does have meaning.)[3]

3.33 In logical syntax the meaning of a sign must never play a role; it must be possible to set up logical syntax without mentioning the *meaning* of a sign; it may presuppose *only* the description of the expressions.

3.331 From this observation we can look across to Russell's "Theory of types": Russell's error shows itself in the fact that in setting up the rules for signs he had to speak of the meaning of the signs.

3.332 No proposition can say anything about itself, because the propositional sign cannot be contained in itself (that is the whole of the "Theory of types").

3.333 A function cannot be its own argument, because the function sign already contains the prototype of its argument, and it cannot contain itself.

For suppose the function F(fx) could be its own argument: then there would be a proposition "F(F(fx))", and in it the outer function F and the inner function F must have different meanings, since the inner one has the form $\phi(fx)$ and the outer one the form $\psi(\phi(fx))$. All the two functions have in common is the letter "F", which by itself signifies nothing.

This becomes clear at once if instead of "F(Fu)" we write "$(\exists\phi):F(\phi u) \cdot \phi u = Fu$".

This disposes of Russell's paradox.

3.334 The rules of logical syntax must go without saying, once we know how each individual sign signifies.

3.34 A proposition has essential and accidental features.

Accidental features are those that result from the particular way in which the propositional sign is produced. Essential features are those that alone enable the proposition to express its sense.

3 William of Ockham (c. 1287–1347), English philosopher; his "razor" holds that entities should not be multiplied beyond necessity.

3.341 So what is essential in a proposition is what all propositions that can express the same sense have in common.

And likewise, in general, what is essential in a symbol is what all symbols that can serve the same purpose have in common.

3.3411 So one could say: the real name is what all symbols that signify the object have in common. It would then follow, step by step, that no kind of composition is essential to a name.

3.342 There is indeed something arbitrary in our notations, but *this* is not arbitrary: that *if* we have determined something arbitrarily, then something else must be the case. (This depends on the *essence* of the notation.)

3.3421 A particular mode of signifying may be unimportant, but it is always important that it is a *possible* mode of signifying. And this is how things are in philosophy generally: again and again the individual case turns out to be unimportant, but the possibility of each individual case reveals something about the essence of the world.

3.343 Definitions are rules for translating from one language into another. Every correct sign-language must be translatable into every other according to such rules: *this* is what they all have in common.

3.344 What signifies in a symbol is what is common to all the symbols by which, according to the rules of logical syntax, it can be replaced.

3.3441 For example, what is common to all notations for truth-functions can be expressed like this: what they have in common is that all of them—for example—*can be replaced* by the notation of "~p" ("not p") and "p $\vee$ q" ("p or q").

(This indicates the way in which a special possible notation can give us general information.)

3.3442 Nor does the sign of a complex resolve arbitrarily in analysis, so that its resolution would be different, say, in every propositional structure.

3.4 A proposition determines a place in logical space. The existence of this logical place is guaranteed by the existence of

the constituents alone, by the existence of the proposition with sense.

3.41 The propositional sign and the logical coordinates: that is the logical place.

3.411 The geometrical place and the logical place agree in that each is the possibility of an existence.

3.42 Although a proposition may determine only one place in logical space, the whole of logical space must nevertheless already be given by it.

(Otherwise negation, the logical sum, the logical product, etc., would keep introducing new elements—in coordination.)

(The logical scaffolding around the picture determines logical space. The proposition reaches through the whole of logical space.)

3.5 The propositional sign, applied and thought, is the thought.

Proposition 4

The thought is the proposition with sense

4 The thought is the proposition with sense.

4.001 The totality of propositions is language.

4.002 Human beings have the ability to construct languages in which every sense can be expressed, without having any idea how or what each word means.—Just as people speak without knowing how the individual sounds are produced.

Everyday language is a part of the human organism and no less complicated than it.

It is humanly impossible to extract the logic of language from it directly.

Language disguises the thought. It does so in such a way that from the outward form of the clothing one cannot infer the form of the thought it clothes; for the outward form of the clothing is designed for quite other purposes than to let the form of the body be recognized.

The tacit conventions on which the understanding of everyday language depends are enormously complicated.

4.003 Most of the propositions and questions that have been written about philosophical matters are not false but nonsensical. So we cannot answer questions of this kind at all; we can only establish that they are nonsensical. Most of the questions and propositions of philosophers arise from our failure to understand the logic of our language.

(They are of the same kind as the question whether the good is more or less identical than the beautiful.)

And it is no wonder that the deepest problems are really *not* problems at all.

4.0031 All philosophy is a "critique of language". (Though not in Mauthner's sense.) It is Russell's achievement to have shown that the apparent logical form of a proposition need not be its real one.[1]

4.01 A proposition is a picture of reality.

A proposition is a model of reality as we think it to be.

4.011 At first sight a proposition—one printed on paper, say—does not seem to be a picture of the reality it is about. But neither does musical notation seem at first sight to be a picture of music, nor our phonetic (alphabetic) script a picture of our spoken language.

And yet these sign-languages turn out to be pictures, even in the ordinary sense, of what they represent.

4.012 It is evident that we perceive a proposition of the form "aRb" as a picture. Here the sign is evidently a likeness of what is signified.

4.013 And if we penetrate to what is essential in this pictorial character, we see that it is *not* disturbed by *apparent irregularities* (such as the use of ♯ and ♭ in musical notation).

For these irregularities, too, depict what they are meant to express; only in a different way.

4.014 The gramophone record, the musical thought, the score, the sound waves all stand to one another in that internal relation of depicting that holds between language and the world.

The logical construction is common to them all.

(Like the two youths in the fairy tale, their two horses, and their lilies. They are all in a certain sense one.)[2]

4.0141 There is a general rule by which the musician can take the symphony from the score, and by which one can derive the symphony from the line on the gramophone record and then, by

1 Fritz Mauthner (1849–1923), Austrian writer whose *Contributions to a Critique of Language* (1901–02) argued for a thoroughgoing skepticism about language.
2 The allusion is to the Grimms' tale "The Gold-Children", in which twin boys, twin horses and two golden lilies are born from the same fish.

the first rule, derive the score again. It is precisely in this that the inner similarity of these seemingly so utterly different constructions consists. And that rule is the law of projection which projects the symphony into the language of musical notation. It is the rule for translating the language of musical notation into the language of the gramophone record.

4.015 The possibility of all likenesses, of the entire pictorial character of our way of expressing ourselves, rests on the logic of depiction.

4.016 To understand the essence of the proposition, think of hieroglyphic writing, which depicts the facts it describes.

And out of it came alphabetic writing, without losing what is essential to depiction.

4.02 We see this from the fact that we understand the sense of a propositional sign without having had it explained to us.

4.021 A proposition is a picture of reality: for if I understand the proposition, I know the situation it represents. And I understand the proposition without having had its sense explained to me.

4.022 A proposition *shows* its sense.

A proposition *shows* how things stand *if* it is true. And it *says that* they do so stand.

4.023 Reality must be pinned down by the proposition to yes or no.

For that, it must be completely described by the proposition.

A proposition is the description of a state of affairs.

Just as a description describes an object by its external properties, so a proposition describes reality by its internal properties.

A proposition constructs a world with the help of a logical scaffolding, and that is why one can also see from the proposition how everything logical stands *if* it is true. One can *draw conclusions* from a false proposition.

4.024 To understand a proposition means to know what is the case if it is true.

(So one can understand it without knowing whether it is true.)

One understands it if one understands its constituents.

4.025 Translating one language into another does not proceed by translating each *proposition* of the one into a *proposition* of the other; only the constituents of propositions are translated.

(And the dictionary translates not only nouns but also verbs, adjectives, conjunctions, etc.; and it treats them all alike.)

4.026 The meanings of simple signs (words) must be explained to us if we are to understand them.

With propositions, however, we communicate.

4.027 It belongs to the essence of a proposition that it can communicate a *new* sense to us.

4.03 A proposition must communicate a new sense with old expressions.

A proposition communicates a situation to us, so it must be *essentially* connected with the situation.

And the connection is precisely that it is its logical picture.

A proposition asserts something only insofar as it is a picture.

4.031 In a proposition a situation is, as it were, put together experimentally.

Instead of saying: this proposition has such-and-such a sense, one can simply say: this proposition represents such-and-such a situation.

4.0311 One name stands for one thing, another for another thing, and they are connected with each other; in this way the whole—like a tableau vivant—presents the state of affairs.

4.0312 The possibility of propositions rests on the principle that signs stand in for objects.

My fundamental thought is that the "logical constants" do not stand in for anything. That nothing can stand in for the *logic* of the facts.

4.032 A proposition is a picture of a situation only insofar as it is logically articulated.

(Even the proposition "ambulo" [I walk] is composite, since its stem with a different ending, or its ending with a different stem, yields a different sense.)

4.04 In a proposition there must be exactly as much to distinguish as in the situation it represents.

The two must have the same logical (mathematical) multiplicity. (Compare Hertz's "Mechanics", on dynamical models.)[3]

4.041 This mathematical multiplicity cannot itself, of course, be depicted in turn. In depicting, one cannot get outside it.

4.0411 Suppose, for example, we wanted to express what we express by "(x).fx" by putting an index in front of "fx"—for instance like this: "Alg.fx". It would not be enough: we would not know what had been generalized. Suppose we wanted to indicate it by a subscript "$_a$"—for instance like this: "f(x_a)". That would not be enough either: we would not know the scope of the generality sign.

Suppose we tried to do it by introducing a marker into the argument places—for instance like this: "(A, A). F(A, A)". It would not be enough: we could not establish the identity of the variables. And so on.

None of these modes of signifying is enough, because they do not have the necessary mathematical multiplicity.[4]

4.0412 For the same reason, the idealist explanation of how we see spatial relations, through "spatial spectacles", is not enough, because it cannot explain the multiplicity of these relations.

4.05 Reality is compared with the proposition.

4.06 A proposition can be true or false only by being a picture of reality.

4.061 If one overlooks the fact that a proposition has a sense independent of the facts, one can easily believe that true and false are relations of equal standing between signs and what they signify.

3 Heinrich Hertz (1857–1894), German physicist; his *Principles of Mechanics* (1894) discusses how one system can serve as a model of another.

4 "Alg." abbreviates German *allgemein*, "general".

One could then say, for example, that "p" signifies in the true way what "~p" signifies in the false way, etc.

4.062 Can we not make ourselves understood with false propositions, as we have so far done with true ones? So long as it is known that they are meant as false? No! For a proposition is true if things are as we say they are by means of it; and if by "p" we mean ~p, and things are as we mean them to be, then in the new interpretation "p" is true and not false.

4.0621 But it is important that the signs "p" and "~p" *can* say the same thing. For this shows that nothing in reality corresponds to the sign "~".

That negation occurs in a proposition is not yet a characteristic of its sense (~~p = p).

The propositions "p" and "~p" have opposite senses, but one and the same reality corresponds to them.

4.063 A picture to explain the concept of truth: a black spot on white paper. One can describe the shape of the spot by stating, for each point on the surface, whether it is white or black. To the fact that a point is black there corresponds a positive fact; to the fact that a point is white (not black), a negative fact. If I designate a point on the surface (a Fregean truth-value), this corresponds to the supposition that is put forward for judgment, etc., etc.

But to be able to say that a point is black or white, I must first know when a point is called black and when it is called white; to be able to say: "p" is true (or false), I must have determined under what circumstances I call "p" true, and in doing so I determine the sense of the proposition.

Now the point where the analogy breaks down is this: we can point to a point on the paper even without knowing what white and black are; but to a proposition without sense nothing at all corresponds, for it does not signify a thing (a truth-value) whose properties might be called "false" or "true". The verb of a proposition is not "is true" or "is false", as Frege thought; rather, what "is true" must already contain the verb.

4.064 Every proposition must *already* have a sense; affirmation cannot give it one, for what it affirms is precisely the sense. And the same holds for negation, etc.

4.0641 One could say: negation already relates to the logical place determined by the negated proposition.

The negating proposition determines a *different* logical place from the negated one.

The negating proposition determines a logical place with the help of the logical place of the negated proposition, by describing it as lying outside the latter.

That one can negate the negated proposition again already shows that what is negated is already a proposition, and not merely the preparation for a proposition.

4.1 A proposition represents the obtaining and non-obtaining of states of affairs.

4.11 The totality of true propositions is the whole of natural science (or the totality of the natural sciences).

4.111 Philosophy is not one of the natural sciences.

(The word "philosophy" must mean something that stands above or below the natural sciences, but not beside them.)

4.112 The aim of philosophy is the logical clarification of thoughts.

Philosophy is not a body of doctrine but an activity.

A philosophical work consists essentially of elucidations.

The result of philosophy is not "philosophical propositions", but the clarification of propositions.

Philosophy should make clear, and sharply delimit, thoughts that are otherwise, as it were, murky and blurred.

4.1121 Psychology is no more closely related to philosophy than any other natural science is.

Theory of knowledge is the philosophy of psychology.

Does not my study of sign-language correspond to the study of thought processes that philosophers held to be so essential to the philosophy of logic? Only they mostly got entangled in inessential psychological investigations, and there is an analogous danger with my method too.

4.1122 Darwin's theory has no more to do with philosophy than any other hypothesis of natural science.

4.113 Philosophy sets limits to the disputable territory of natural science.

4.114 It should delimit the thinkable, and with it the unthinkable.

It should limit the unthinkable from within, by means of the thinkable.

4.115 It will signify what cannot be said by representing clearly what can be said.

4.116 Everything that can be thought at all can be thought clearly. Everything that can be put into words can be put into words clearly.

4.12 A proposition can represent the whole of reality, but it cannot represent what it must have in common with reality in order to be able to represent it—logical form.

To be able to represent logical form, we would have to be able to station ourselves, together with the proposition, outside logic —that is, outside the world.

4.121 A proposition cannot represent logical form; logical form is mirrored in it.

What is mirrored in language, language cannot represent.

What expresses *itself* in language, *we* cannot express by means of language.

A proposition *shows* the logical form of reality.

It displays it.

4.1211 Thus a proposition "fa" shows that the object a occurs in its sense; two propositions "fa" and "ga" show that they are both about the same object.

If two propositions contradict each other, their structure shows it; likewise if one follows from the other. And so on.

4.1212 What *can* be shown *cannot* be said.

4.1213 Now we also understand our feeling that we possess a correct logical conception once everything in our sign-language is in order.

4.122 In a certain sense we can speak of formal properties of objects and states of affairs, or of properties of the structure of facts, and in the same sense of formal relations and of relations of structures.

(Instead of "property of the structure" I also say "internal property"; instead of "relation of the structures", "internal relation".

I introduce these expressions in order to show the source of the confusion, very widespread among philosophers, between internal relations and relations proper (external relations).)

The obtaining of such internal properties and relations cannot, however, be asserted by propositions; rather, it shows itself in the propositions that represent those states of affairs and are about those objects.

4.1221 An internal property of a fact can also be called a feature of that fact. (In the sense in which we speak of facial features, for example.)

4.123 A property is internal if it is unthinkable that its object should not possess it.

(This shade of blue and that one stand, eo ipso [by that very fact], in the internal relation of lighter and darker. It is unthinkable that *these* two objects should not stand in this relation.)

(Here the shifting use of the words "property" and "relation" corresponds to the shifting use of the word "object".)

4.124 The obtaining of an internal property of a possible situation is not expressed by a proposition; rather, it expresses itself in the proposition that represents the situation, through an internal property of that proposition.

It would be just as nonsensical to ascribe a formal property to a proposition as to deny it that property.

4.1241 Forms cannot be distinguished from one another by saying that one has this property and the other that one; for this presupposes that it makes sense to assert either property of either form.

4.125 The obtaining of an internal relation between possible situations is expressed in language by an internal relation between the propositions that represent them.

4.1251 This now settles the disputed question "whether all relations are internal or external".

4.1252 Series that are ordered by *internal* relations I call series of forms.

The number series is ordered not by an external relation but by an internal one.

Likewise the series of propositions "aRb",

"(∃x):aRx . xRb",

"(∃x,y):aRx . xRy . yRb", and so on.

(If b stands in one of these relations to a, I call b a successor of a.)

4.126 In the sense in which we speak of formal properties, we can now also speak of formal concepts.

(I introduce this expression in order to make clear the source of the confusion between formal concepts and concepts proper, which runs through the whole of the old logic.)

That something falls under a formal concept as its object cannot be expressed by a proposition. Instead, it shows itself in the sign for this object itself. (A name shows that it signifies an object, a numeral that it signifies a number, etc.)

Formal concepts, after all, cannot be represented by a function, as concepts proper can.

For their characteristics, the formal properties, are not expressed by functions.

The expression of a formal property is a feature of certain symbols.

So the sign for the characteristics of a formal concept is a distinctive feature of all symbols whose meanings fall under the concept.

So the expression of a formal concept is a propositional variable in which only this distinctive feature is constant.

4.127 The propositional variable signifies the formal concept, and its values signify the objects that fall under this concept.

4.1271 Every variable is the sign of a formal concept.

For every variable represents a constant form that all its values possess, and that can be construed as a formal property of these values.

4.1272 Thus the variable name "x" is the proper sign of the pseudo-concept *object*.

Wherever the word "object" ("thing", "item", etc.) is used correctly, it is expressed in concept-script by the variable name.

For example, in the proposition "there are 2 objects which ..." by "(∃x,y) ...".

Wherever it is used otherwise, that is, as a concept-word proper, nonsensical pseudo-propositions arise.

So one cannot say, for example, "There are objects", as one might say "There are books". Nor, likewise, "There are 100 objects" or "There are $\aleph_0$ objects".

And it is nonsensical to speak of the *number of all objects*.

The same holds for the words "complex", "fact", "function", "number", etc.

They all signify formal concepts and are represented in concept-script by variables, not by functions or classes. (As Frege and Russell believed.)

Expressions such as "1 is a number", "there is only one zero", and all similar ones are nonsensical.

(It is just as nonsensical to say "there is only one 1" as it would be to say: 2 + 2 at 3 o'clock equals 4.)

4.12721 A formal concept is already given with an object that falls under it. So one cannot introduce both objects of a formal concept *and* the formal concept itself as primitive concepts. So one cannot, for example, introduce as primitive concepts both the concept of a function and particular functions (as Russell does); or the concept of number and particular numbers.

4.1273 If we want to express the general proposition "b is a successor of a" in concept-script, we need for this an expression for the general term of the series of forms:

aRb,

(∃x):aRx . xRb,

(∃x,y):aRx . xRy . yRb,

… .

The general term of a series of forms can be expressed only by a variable, because the concept "term of this series of forms" is a *formal* concept. (Frege and Russell overlooked this; the way in which they want to express general propositions like the one above is therefore wrong; it contains a circulus vitiosus [vicious circle].)

We can determine the general term of the series of forms by giving its first term and the general form of the operation that generates the next term from the preceding proposition.

4.1274 The question whether a formal concept exists is nonsensical. For no proposition can answer such a question.

(So one cannot ask, for example: "Are there unanalyzable subject–predicate propositions?")

4.128 Logical forms are number*less*.

That is why there are no privileged numbers in logic, and why there is no philosophical monism or dualism, etc.

4.2 The sense of a proposition is its agreement and disagreement with the possibilities of the obtaining and non-obtaining of states of affairs.

4.21 The simplest proposition, the elementary proposition, asserts the obtaining of a state of affairs.

4.211 It is a sign of an elementary proposition that no elementary proposition can contradict it.

4.22 An elementary proposition consists of names. It is a nexus, a concatenation, of names.

4.221 It is obvious that in analyzing propositions we must arrive at elementary propositions consisting of names in immediate combination.

The question here is how the propositional bond comes about.

4.2211 Even if the world is infinitely complex, so that every fact consists of infinitely many states of affairs and every state of affairs is composed of infinitely many objects, even then there would have to be objects and states of affairs.

4.23 A name occurs in a proposition only in the context of an elementary proposition.

4.24 Names are the simple symbols; I indicate them by single letters (“x”, “y”, “z”).

I write the elementary proposition as a function of names, in the form “fx”, “φ(x,y)”, etc.

Or I indicate it by the letters p, q, r.

4.241 If I use two signs with one and the same meaning, I express this by putting the sign “=” between them.

So “a = b” means: the sign “a” can be replaced by the sign “b”.

(If I introduce a new sign “b” by means of an equation, stipulating that it is to replace an already known sign “a”, I write the equation—the definition—(as Russell does) in the form “a = b Def.”. A definition is a rule about signs.)

4.242 So expressions of the form “a = b” are merely aids to representation; they say nothing about the meaning of the signs “a” and “b”.

4.243 Can we understand two names without knowing whether they signify the same thing or two different things?—Can we understand a proposition in which two names occur without knowing whether they mean the same or something different?

If I know, say, the meaning of an English word and of a German word that means the same, it is impossible for me not to know that the two mean the same; it is impossible for me not to be able to translate them into each other.

Expressions like “a = a”, or ones derived from them, are neither elementary propositions nor signs with sense of any other kind. (This will show itself later.)

4.25 If the elementary proposition is true, the state of affairs obtains; if the elementary proposition is false, the state of affairs does not obtain.

4.26 Specifying all true elementary propositions describes the world completely. The world is completely described by specifying all elementary propositions plus specifying which of them are true and which are false.

4.27 With regard to the obtaining and non-obtaining of n states of affairs, there are $K_n = \sum_{\nu=0}^{n} (n\ \nu)$ possibilities.

Any combination of the states of affairs can obtain while the others do not obtain.

4.28 To these combinations correspond the same number of possibilities of truth—and falsity—of n elementary propositions.

4.3 The truth-possibilities of elementary propositions mean the possibilities of the obtaining and non-obtaining of states of affairs.

4.31 We can represent the truth-possibilities by schemata of the following kind ("W" means "true", "F" "false". The rows of "W"s and "F"s under the row of elementary propositions signify their truth-possibilities in an easily understood symbolism):

p q r
W W W
F W W
W F W
W W F
F F W
F W F
W F F
F F F

p q
W W
F W
W F
F F

p
W
F[5]

4.4 A proposition is the expression of agreement and disagreement with the truth-possibilities of elementary propositions.

4.41 The truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions.

5 "W" and "F" abbreviate German *wahr* (true) and *falsch* (false); the original notation is kept throughout.

4.411 It is likely from the outset that the introduction of elementary propositions is fundamental to the understanding of all other kinds of proposition. Indeed, the understanding of general propositions *palpably* depends on that of elementary propositions.

4.42 With regard to a proposition's agreement and disagreement with the truth-possibilities of n elementary propositions, there are $\sum_{\kappa=0}^{K_n} (K_n \kappa) = L_n$ possibilities.

4.43 We can express agreement with truth-possibilities by assigning to them in the schema, say, the mark "W" (true).

The absence of this mark means disagreement.

4.431 The expression of agreement and disagreement with the truth-possibilities of elementary propositions expresses the truth-conditions of the proposition.

A proposition is the expression of its truth-conditions.

(Frege was therefore quite right to put them first, as an explanation of the signs of his concept-script. Only Frege's explanation of the concept of truth is wrong: if "the True" and "the False" really were objects, and were the arguments in ~p etc., then by Frege's own stipulation the sense of "~p" would not be determined at all.)

4.44 The sign that results from correlating those marks "W" with the truth-possibilities is a propositional sign.

4.441 It is clear that no object (or complex of objects) corresponds to the complex of the signs "F" and "W", any more than to the horizontal and vertical lines or to the brackets.—There are no "logical objects".

Something analogous of course holds for all signs that express the same as the schemata of "W"s and "F"s.

4.442 For example,

"p q
W W W
F W W
W F
F F W"
is a propositional sign.

(Frege's "judgment stroke" "⊢" is logically quite meaningless; in Frege (and Russell) it merely indicates that these authors hold the propositions so marked to be true. "⊢" therefore belongs to the structure of the proposition no more than, say, the proposition's number does. A proposition cannot possibly say of itself that it is true.)

If the order of the truth-possibilities in the schema is fixed once and for all by a rule of combination, then the last column by itself is already an expression of the truth-conditions. If we write this column out as a row, the propositional sign becomes

"(WW–W) (p, q)"

or, more clearly,

"(WWFW) (p, q)".

(The number of places in the left-hand bracket is determined by the number of terms in the right-hand one.)

4.45 For n elementary propositions there are L_n possible groups of truth-conditions.

The groups of truth-conditions that belong to the truth-possibilities of a given number of elementary propositions can be arranged in a series.

4.46 Among the possible groups of truth-conditions there are two extreme cases.

In the one case the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth-conditions are *tautological*.

In the second case the proposition is false for all the truth-possibilities: the truth-conditions are *contradictory*.

In the first case we call the proposition a tautology, in the second a contradiction.

4.461 A proposition shows what it says; a tautology and a contradiction show that they say nothing.

A tautology has no truth-conditions, since it is unconditionally true; and a contradiction is true under no condition.

Tautology and contradiction are senseless.

(Like the point from which two arrows go out in opposite directions.)

(For example, I know nothing about the weather when I know that it is either raining or not raining.)

4.4611 Tautology and contradiction are not, however, nonsensical; they belong to the symbolism, much as "0" belongs to the symbolism of arithmetic.

4.462 Tautology and contradiction are not pictures of reality. They do not represent any possible situation. For the one admits *every* possible situation, the other *none*.

In a tautology the conditions of agreement with the world—the representing relations—cancel one another out, so that it stands in no representing relation to reality.

4.463 The truth-conditions determine the range left open to the facts by the proposition.

(A proposition, a picture, a model is, in the negative sense, like a solid body that restricts the freedom of movement of others; in the positive sense, like the space bounded by solid substance, in which a body has room.)

A tautology leaves to reality the whole—infinite—logical space; a contradiction fills the whole of logical space and leaves reality not a single point. So neither of them can determine reality in any way.

4.464 The truth of a tautology is certain, that of a proposition possible, that of a contradiction impossible.

(Certain, possible, impossible: here we have the first indication of the gradation that we need in the theory of probability.)

4.465 The logical product of a tautology and a proposition says the same as the proposition. So that product is identical with the proposition. For one cannot alter what is essential to a symbol without altering its sense.

4.466 To a definite logical combination of signs corresponds a definite logical combination of their meanings; *any arbitrary* combination corresponds only to the uncombined signs.

That is, propositions that are true for every situation cannot be combinations of signs at all, for otherwise only definite combinations of objects could correspond to them.

(And to no logical combination there corresponds *no* combination of the objects.)

Tautology and contradiction are the limiting cases of the combination of signs—namely, its dissolution.

4.4661 Admittedly, in tautology and contradiction too the signs are still combined with one another, that is, they stand in relations to one another; but these relations are meaningless, inessential to the *symbol*.

4.5 It now seems possible to give the most general propositional form: that is, to give a description of the propositions of *any* sign-language whatever, such that every possible sense can be expressed by a symbol that fits the description, and every symbol that fits the description can express a sense, provided the meanings of the names are suitably chosen.

It is clear that in describing the most general propositional form *only* what is essential to it may be described—otherwise it would not be the most general.

That there is a general propositional form is proved by the fact that there cannot be a proposition whose form could not have been foreseen (that is, constructed). The general form of the proposition is: Things are thus and so.

4.51 Suppose I were given *all* elementary propositions: then one can simply ask: What propositions can I form from them? And these are *all* propositions, and *that* is how they are bounded.

4.52 Propositions are everything that follows from the totality of all elementary propositions (and of course also from its being the *totality of all* of them). (So one could say, in a certain sense, that *all* propositions are generalizations of elementary propositions.)

4.53 The general propositional form is a variable.

Proposition 5

Propositions are truth-functions of elementary propositions

5 A proposition is a truth-function of elementary propositions.

(An elementary proposition is a truth-function of itself.)

5.01 Elementary propositions are the truth-arguments of a proposition.

5.02 It is tempting to confuse the arguments of functions with the indices of names. For I recognize the meaning of the sign containing them from the argument just as I do from the index.

In Russell's "+_{c}", for example, "c" is an index pointing to the fact that the whole sign is the addition sign for cardinal numbers. But this designation rests on arbitrary convention, and one could choose a simple sign instead of "+_{c}". In "~p", however, "p" is not an index but an argument: the sense of "~p" *cannot* be understood unless the sense of "p" has been understood first. (In the name Julius Caesar, "Julius" is an index. An index is always part of a description of the object to whose name we attach it. For example, *the* Caesar of the Julian clan.)

If I am not mistaken, the confusion of argument and index underlies Frege's theory of the meaning of propositions and functions. For Frege the propositions of logic were names, and their arguments the indices of these names.

5.1 Truth-functions can be ordered in series.

That is the foundation of the theory of probability.

5.101 The truth-functions of any number of elementary propositions can be written out in a schema of the following kind:

(WWWW) (p, q) Tautology (If p then p; and if q then q.) (p ⊃ p . q ⊃ q)

(FWWW) (p, q) in words: Not both p and q. (~(p . q))
(WFWW) (p, q) " " If q then p. (q ⊃ p)
(WWFW) (p, q) " " If p then q. (p ⊃ q)
(WWWF) (p, q) " " p or q. (p ∨ q)
(FFWW) (p, q) " " Not q. (~q)
(FWFW) (p, q) " " Not p. (~p)
(FWWF) (p, q) " " p or q, but not both. (p . ~q :∨: q . ~p)
(WFFW) (p, q) " " If p then q; and if q then p. (p ≡ q)
(WFWF) (p, q) " " p
(WWFF) (p, q) " " q
(FFFW) (p, q) " " Neither p nor q. (~p . ~q) or (p | q)
(FFWF) (p, q) " " p and not q. (p . ~q)
(FWFF) (p, q) " " q and not p. (q . ~p)
(WFFF) (p, q) " " q and p. (q . p)
(FFFF) (p, q) Contradiction (p and not p; and q and not q.) (p . ~p . q . ~q)

Those truth-possibilities of its truth-arguments that make a proposition true I shall call its *truth-grounds*.

5.11 If all the truth-grounds common to a number of propositions are also truth-grounds of a certain proposition, we say that the truth of that proposition follows from the truth of those propositions.

5.12 In particular, the truth of a proposition "p" follows from the truth of another, "q", if all the truth-grounds of the second are truth-grounds of the first.

5.121 The truth-grounds of the one are contained in those of the other; p follows from q.

5.122 If p follows from q, the sense of "p" is contained in the sense of "q".

5.123 If a god creates a world in which certain propositions are true, then in doing so he already creates a world in which all their consequences hold. And similarly he could not create a world in which the proposition "p" is true without creating all its objects.

5.124 A proposition affirms every proposition that follows from it.

5.1241 “p . q” is one of the propositions that affirm “p”, and at the same time one of the propositions that affirm “q”.

Two propositions are opposed to each other if there is no proposition with sense that affirms them both.

Every proposition that contradicts another denies it.

5.13 That the truth of a proposition follows from the truth of other propositions is something we see from the structure of the propositions.

5.131 If the truth of a proposition follows from the truth of others, this is expressed by relations in which the forms of those propositions stand to one another. Nor do we have to put them into those relations first by combining them with one another in a proposition; rather, these relations are internal, and they obtain as soon as, and simply because, the propositions exist.

5.1311 When we infer q from p $\vee$ q and ~p, the relation between the propositional forms of “p $\vee$ q” and “~p” is concealed here by the way of symbolizing. But if, for example, we write “p | q .|. p | q” instead of “p $\vee$ q”, and “p | p” instead of “~p” (p | q = neither p nor q), then the inner connection becomes obvious.

(That we can infer fa from (x).fx shows that generality is present in the symbol “(x).fx” too.)

5.132 If p follows from q, I can infer p from q; deduce p from q.

The kind of inference can be gathered from the two propositions alone.

Only they themselves can justify the inference.

“Laws of inference”, which—as in Frege and Russell—are supposed to justify inferences, are senseless, and would be superfluous.

5.133 All deduction takes place a priori.

5.134 From one elementary proposition no other can be deduced.

5.135 In no way can we infer from the obtaining of any one situation the obtaining of another situation entirely different from it.

5.136 There is no causal nexus that would justify such an inference.

5.1361 We *cannot* infer the events of the future from those of the present.

Belief in the causal nexus is *superstition*.

5.1362 The freedom of the will consists in the fact that future actions cannot be known now. We could know them only if causality were an *inner* necessity, like that of logical inference.—The connection between knowing and what is known is that of logical necessity.

("A knows that p is the case" is senseless if p is a tautology.)

5.1363 If it does not *follow* from a proposition's being self-evident to us that it is true, then its self-evidence is no justification either for our belief in its truth.

5.14 If one proposition follows from another, the latter says more than the former, and the former less than the latter.

5.141 If p follows from q and q from p, then they are one and the same proposition.

5.142 A tautology follows from all propositions: it says nothing.

5.143 Contradiction is what propositions have in common that *no* proposition has in common with another. Tautology is what all propositions have in common that have nothing in common with one another.

Contradiction vanishes, so to speak, outside all propositions, tautology inside them.

Contradiction is the outer limit of propositions; tautology is their center, without substance.

5.15 If W_r is the number of truth-grounds of the proposition "r", and W_{rs} the number of those truth-grounds of the proposition "s" that are also truth-grounds of "r", then we call the ratio $W_{rs} : W_r$ the measure of the *probability* that the proposition "r" gives to the proposition "s".

5.151 In a schema like the one above in No. 5.101, let W_r be the number of "W"s in the proposition r, and W_{rs} the number of those "W"s in the proposition s that stand in the same columns as "W"s of the proposition r. The proposition r then gives the proposition s the probability $W_{rs} : W_r$.

5.1511 There is no special object peculiar to probability propositions.

5.152 Propositions that have no truth-arguments in common with one another we call independent of one another.

Two elementary propositions give each other the probability ½.

If p follows from q, then the proposition "q" gives the proposition "p" the probability 1. The certainty of logical inference is a limiting case of probability.

(Application to tautology and contradiction.)

5.153 A proposition is in itself neither probable nor improbable. An event occurs or it does not occur; there is no middle course.

5.154 Suppose an urn contains equal numbers of white and black balls (and no others). I draw one ball after another and put each back into the urn. Then I can establish by experiment that, as I go on drawing, the numbers of black and white balls drawn come closer to each other.

So *that* is not a mathematical fact.

Now if I say, "It is equally probable that I will draw a white ball as a black one", this means: all the circumstances known to me (including the laws of nature assumed as hypotheses) give the occurrence of the one event no *more* probability than the occurrence of the other. That is to say, they give each—as can easily be gathered from the explanations above—the probability ½.

What I confirm by the experiment is that the occurrence of the two events is independent of the circumstances of which I have no closer knowledge.

5.155 The unit of the probability proposition is: the circumstances—of which I have no further knowledge—give the occurrence of a particular event such-and-such a degree of probability.

5.156 So probability is a generalization.

It involves a general description of a propositional form.

Only when we lack certainty do we make use of probability.—When we do not know a fact completely, but do know *something* about its form.

(A proposition may well be an incomplete picture of a certain situation, but it is always *a* complete picture.)

A probability proposition is, as it were, an excerpt from other propositions.

5.2 The structures of propositions stand in internal relations to one another.

5.21 We can bring out these internal relations in our way of expressing things by representing a proposition as the result of an operation that produces it from other propositions (the bases of the operation).

5.22 An operation is the expression of a relation between the structures of its result and of its bases.

5.23 An operation is what has to be done to one proposition in order to make the other out of it.

5.231 And that will of course depend on their formal properties, on the internal similarity of their forms.

5.232 The internal relation that orders a series is equivalent to the operation by which one term arises from another.

5.233 An operation can only come in where one proposition arises from another in a logically meaningful way. That is, where the logical construction of the proposition begins.

5.234 The truth-functions of elementary propositions are results of operations that have the elementary propositions as bases. (I call these operations truth-operations.)

5.2341 The sense of a truth-function of p is a function of the sense of p.

Negation, logical addition, logical multiplication, etc., etc. are operations.

(Negation reverses the sense of a proposition.)

5.24 An operation shows itself in a variable; it shows how we can get from one form of proposition to another.

It gives expression to the difference between the forms.

(And what the bases have in common with the result of the operation is precisely the bases.)

5.241 An operation does not characterize a form, but only the difference between forms.

5.242 The same operation that makes "q" from "p" makes "r" from "q", and so on. This can only be expressed by the fact that "p", "q", "r", etc. are variables that give general expression to certain formal relations.

5.25 The occurrence of an operation does not characterize the sense of a proposition.

After all, an operation asserts nothing; only its result does, and this depends on the bases of the operation.

(Operation and function must not be confused with each other.)

5.251 A function cannot be its own argument, but the result of an operation can very well become its own base.

5.252 Only in this way is it possible to advance from term to term in a series of forms (from type to type in the hierarchies of Russell and Whitehead). (Russell and Whitehead did not admit the possibility of this advance, yet they made use of it again and again.)[1]

5.2521 The repeated application of an operation to its own result I call its successive application ("O'O'O'a" is the result of applying "O'ξ" successively three times to "a").

In a similar sense I speak of the successive application of *several* operations to a number of propositions.

5.2522 Accordingly, I write the general term of a series of forms a, O'a, O'O'a, ... like this: "[a, x, O'x]". This bracketed expression is a variable. The first term of the bracketed expression is the beginning of the series of forms, the second is the form of an arbitrary term x of the series, and the third is the form of the term of the series that immediately follows x.

5.2523 The concept of the successive application of an operation is equivalent to the concept "and so on".

1 Alfred North Whitehead (1861–1947), co-author with Russell of *Principia Mathematica* (1910–13), whose theory of types arranges logical entities in a hierarchy.

5.253 One operation can undo the effect of another. Operations can cancel each other out.

5.254 An operation can vanish (for example, negation in "~~p": ~~p = p).

5.3 All propositions are results of truth-operations on elementary propositions.

A truth-operation is the way in which a truth-function arises from elementary propositions.

By the very nature of truth-operations, a new truth-function arises from truth-functions in the same way as their truth-function arises from elementary propositions. Every truth-operation produces from truth-functions of elementary propositions another truth-function of elementary propositions, a proposition. The result of every truth-operation on the results of truth-operations on elementary propositions is again the result of *one* truth-operation on elementary propositions.

Every proposition is the result of truth-operations on elementary propositions.

5.31 The schemata in No. 4.31 have a meaning even when "p", "q", "r", etc. are not elementary propositions.

And it is easy to see that the propositional sign in No. 4.442 expresses a single truth-function of elementary propositions even when "p" and "q" are truth-functions of elementary propositions.

5.32 All truth-functions are results of the successive application of a finite number of truth-operations to elementary propositions.

5.4 Here it shows itself that there are no "logical objects" or "logical constants" (in the sense of Frege and Russell).

5.41 For all the results of truth-operations on truth-functions that are one and the same truth-function of elementary propositions are identical.

5.42 That $\vee$, $\supset$, etc. are not relations in the sense in which right and left etc. are relations is obvious.

The fact that Frege's and Russell's logical "primitive signs" can be defined in terms of one another already shows that they

are not primitive signs, and still more that they do not signify relations.

And it is plain that the "⊃" we define by means of "~" and "∨" is identical with the one by means of which we define "∨" with "~", and that this "∨" is identical with the first. And so on.

5.43 That from one fact p infinitely many *others* should follow, namely ~~p, ~~~~p, etc., is surely hard to believe from the outset. And it is no less remarkable that the infinite number of propositions of logic (of mathematics) should follow from half a dozen "basic laws".

But all the propositions of logic say the same thing. Namely, nothing.

5.44 Truth-functions are not material functions.

If, for example, an affirmation can be produced by double negation, is negation then—in any sense—contained in affirmation? Does "~~p" deny ~p, or does it affirm p; or both?

The proposition "~~p" is not about negation as if it were an object; but the possibility of negation is already prejudged in affirmation.

And if there were an object called "~", then "~~p" would have to say something different from "p". For the one proposition would then be about ~, and the other would not.

5.441 This vanishing of the apparent logical constants also occurs if "~(∃x).~fx" says the same as "(x).fx", or "(∃x).fx . x = a" the same as "fa".

5.442 If we are given a proposition, then *with it* we are also already given the results of all truth-operations that have it as their base.

5.45 If there are logical primitive signs, a correct logic must make clear their position relative to one another and justify their existence. The construction of logic *out of* its primitive signs must become clear.

5.451 If logic has primitive concepts, they must be independent of one another. If a primitive concept has been introduced, it must have been introduced in all the combinations in which it occurs at all. So one cannot introduce it first for *one* combina-

tion and then again for another. For example, once negation has been introduced, we must understand it in propositions of the form "~p" just as we understand it in propositions like "~(p ∨ q)", "(∃x).~fx" and others. We must not introduce it first for one class of cases and then for another, for it would then remain doubtful whether its meaning was the same in both cases, and there would be no reason to use the same way of combining signs in both.

(In short, what Frege said (in "Grundgesetze der Arithmetik") about introducing signs by definitions holds, mutatis mutandis, for introducing primitive signs as well.)[2]

5.452 Introducing a new device into the symbolism of logic must always be a momentous event. No new device may be slipped into logic in brackets or in a footnote—with an entirely innocent air, so to speak.

(Thus in Russell and Whitehead's "Principia Mathematica" definitions and basic laws occur in words. Why suddenly words here? This would need a justification. There is none, and there can be none, since the procedure is in fact illegitimate.)

But if introducing a new device has proved necessary at one point, we must immediately ask: where must this device now *always* be used? Its position in logic must now be explained.

5.453 All numbers in logic must be capable of justification.

Or rather: it must become evident that there are no numbers in logic.

There are no privileged numbers.

5.454 In logic there is no side by side; there can be no classification.

In logic there cannot be the more general and the more specific.

5.4541 The solutions of logical problems must be simple, since they set the standard of simplicity.

2 Frege's *Basic Laws of Arithmetic* (1893–1903), his attempt to derive arithmetic from logic.

People have always sensed that there must be a domain of questions whose answers—a priori—are symmetrical and lie united in a closed, regular whole.

A domain in which the principle holds: simplex sigillum veri [simplicity is the seal of truth].

5.46 If we introduced the logical signs properly, we would thereby already have introduced the sense of all their combinations; that is, not only "p ∨ q" but also "~(p ∨ ~q)" etc. etc. We would thereby also already have introduced the effect of all possible combinations of brackets. And that would have made it clear that the real general primitive signs are not "p ∨ q", "(∃x).fx", etc., but the most general form of their combinations.

5.461 The apparently unimportant fact that the logical pseudo-relations, such as ∨ and ⊃, need brackets—unlike real relations—is significant.

Indeed, the use of brackets with those apparent primitive signs already indicates that they are not the real primitive signs. And surely no one will believe that the brackets have a meaning of their own.

5.4611 Signs for logical operations are punctuation marks.

5.47 It is clear that whatever can be said *in advance* about the form of all propositions at all must be sayable *all at once*.

After all, all logical operations are already contained in the elementary proposition. For "fa" says the same as

"(∃x).fx . x = a".

Where there is compositeness, there are argument and function, and where these are, all the logical constants already are.

One could say: the one logical constant is what *all* propositions, by their nature, have in common with one another.

But that is the general propositional form.

5.471 The general propositional form is the essence of a proposition.

5.4711 To give the essence of a proposition means to give the essence of all description, and so the essence of the world.

5.472 The description of the most general propositional form is the description of the one and only general primitive sign of logic.

5.473 Logic must take care of itself.

A *possible* sign must also be able to signify. Everything that is possible in logic is also permitted. ("Socrates is identical" means nothing because there is no property called "identical". The proposition is nonsensical because we have not made some arbitrary stipulation, not because the symbol in and of itself would be impermissible.)

In a certain sense, we cannot make mistakes in logic.

5.4731 The self-evidence Russell talked about so much can become dispensable in logic only because language itself prevents every logical error.—That logic is a priori consists in the fact that we *cannot* think illogically.

5.4732 We cannot give a sign the wrong sense.

5.47321 Occam's maxim is of course not an arbitrary rule, nor one justified by its practical success: it says that *unnecessary* units in a sign-language mean nothing.

Signs that serve *one* purpose are logically equivalent; signs that serve *no* purpose are logically meaningless.

5.4733 Frege says: every legitimately constructed proposition must have a sense. And I say: every possible proposition is legitimately constructed, and if it has no sense, that can only be because we have given no *meaning* to some of its constituents.

(Even if we believe we have done so.)

Thus "Socrates is identical" says nothing because we have given the word "identical" *no* meaning as an *adjective*. For when it occurs as the sign of equality, it symbolizes in an entirely different way—the signifying relation is a different one—and so the symbol is entirely different in the two cases too; the two symbols merely happen to have the sign in common.

5.474 The number of fundamental operations needed depends *only* on our notation.

5.475 All that matters is to construct a system of signs with a particular number of dimensions—with a particular mathematical multiplicity.

5.476 It is clear that what is at issue here is not a *number of primitive concepts* that have to be signified, but the expression of a rule.

5.5 Every truth-function is a result of the successive application of the operation (−−−−−W) (ξ, ...) to elementary propositions.

This operation negates all the propositions in the right-hand bracket, and I call it the negation of those propositions.

5.501 A bracketed expression whose terms are propositions I indicate—if the order of the terms in the bracket does not matter —by a sign of the form "(ξ̄)". "ξ̄" is a variable whose values are the terms of the bracketed expression; and the bar over the variable indicates that it stands in for all its values in the bracket.

(So if ξ̄ has, say, the 3 values P, Q, R, then (ξ̄) = (P, Q, R).)

The values of the variable are stipulated.

The stipulation is the description of the propositions that the variable stands in for.

How the description of the terms of the bracketed expression is given is not essential.

We *can* distinguish three kinds of description: 1. Direct enumeration. In this case we can simply put its constant values in place of the variable. 2. Giving a function fx whose values for all values of x are the propositions to be described. 3. Giving a formal law according to which those propositions are constructed. In this case the terms of the bracketed expression are all the terms of a series of forms.

5.502 So instead of "(−−−−−W) (ξ, ...)" I write "N(ξ̄)".

N(ξ̄) is the negation of all the values of the propositional variable ξ̄.

5.503 Since it is evidently easy to express how propositions can be constructed with this operation and how they cannot be constructed with it, this must also be capable of exact expression.

5.51 If ξ has only one value, then N(ξ) = ~p (not p); if it has two values, then N (ξ) = ~p . ~q (neither p nor q).

5.511 How can all-embracing logic, which mirrors the world, use such peculiar hooks and manipulations? Only because they all join together into an infinitely fine network, into the great mirror.

5.512 "~p" is true if "p" is false. So in the true proposition "~p", "p" is a false proposition. How then can the stroke "~" make it agree with reality?

But what does the negating in "~p" is not the "~"; it is what all the signs of this notation that negate p have in common.

That is, the common rule according to which "~p", "~~~p", "~p ∨ ~p", "~p . ~p", etc. etc. (ad inf.) are constructed. And this common factor mirrors negation.

5.513 One could say: what all symbols that affirm both p and q have in common is the proposition "p . q". What all symbols that affirm either p or q have in common is the proposition "p ∨ q".

And so one can say: two propositions are opposed to each other if they have nothing in common with each other; and: every proposition has only one negative, because there is only one proposition that lies entirely outside it.

Thus it also shows itself in Russell's notation that "q : p ∨ ~p" says the same as "q"; that "p ∨ ~p" says nothing.

5.514 Once a notation has been fixed, there is in it a rule according to which all propositions negating p are constructed, a rule according to which all propositions affirming p are constructed, a rule according to which all propositions affirming p or q are constructed, and so on. These rules are equivalent to the symbols, and in them their sense is mirrored.

5.515 It must show itself in our symbols that what is combined by "∨", ".", etc. must be propositions.

And this is indeed the case, for the symbols "p" and "q" themselves presuppose "∨", "~", etc. If the sign "p" in "p ∨ q" does not stand for a complex sign, then it cannot have sense on its own; but then the signs with the same sense as "p", namely "p ∨

p", "p . p", etc., cannot have sense either. And if "p ∨ p" has no sense, then "p ∨ q" cannot have sense either.

5.5151 Must the sign of a negative proposition be constructed with the sign of the positive one? Why should it not be possible to express a negative proposition by means of a negative fact? (For example: if "a" does not stand in a certain relation to "b", that could express that aRb is not the case.)

But here too the negative proposition is, after all, constructed indirectly by means of the positive.

A positive *proposition* must presuppose the existence of the negative *proposition*, and vice versa.

5.52 If the values of ξ are all the values of a function fx for all values of x, then N(ξ) = ~(∃x).fx.

5.521 I separate the concept *all* from the truth-function.

Frege and Russell introduced generality in connection with the logical product or the logical sum. This made it hard to understand the propositions "(∃x).fx" and "(x).fx", in which both ideas are embedded.

5.522 What is peculiar to the notation for generality is, first, that it points to a logical prototype and, second, that it gives prominence to constants.

5.523 The notation for generality occurs as an argument.

5.524 If objects are given, then *all* objects are thereby already given to us.

If elementary propositions are given, then *all* elementary propositions are thereby given too.

5.525 It is incorrect to render the proposition "(∃x).fx" in words —as Russell does—by "fx is *possible*".

Certainty, possibility or impossibility of a situation is not expressed by a proposition, but by an expression's being a tautology, a proposition with sense, or a contradiction.

The precedent to which one would always like to appeal must already lie in the symbol itself.

5.526 The world can be completely described by fully generalized propositions, that is, without assigning any name to a particular object in advance.

Then, to arrive at the ordinary way of expressing things, one simply has to say, after an expression "There is one and only one x such that …": And this x is a.

5.5261 A fully generalized proposition is composite, like every other proposition. (This shows itself in the fact that in "(∃x, ϕ).ϕx" we have to mention "ϕ" and "x" separately. Both stand independently in signifying relations to the world, as in the ungeneralized proposition.)

Mark of a composite symbol: it has something in common with *other* symbols.

5.5262 For the truth or falsity of *every* proposition changes something in the general construction of the world. And the room left to its construction by the totality of elementary propositions is precisely the room that the wholly general propositions delimit.

(If an elementary proposition is true, then at any rate one *more* elementary proposition is true.)

5.53 Identity of object I express by identity of sign, and not by means of a sign of identity. Difference of objects by difference of signs.

5.5301 That identity is not a relation between objects is obvious. This becomes very clear if one considers, for example, the proposition "(x):fx .ɔ. x = a". What this proposition says is simply that *only* a satisfies the function f, and not that only things that have a certain relation to a satisfy the function f.

One could of course say that *only* a has this relation to a; but to express that, we would need the sign of identity itself.

5.5302 Russell's definition of "=" is inadequate, because according to it one cannot say that two objects have all their properties in common. (Even if this proposition is never true, it still has *sense*.)

5.5303 Incidentally: to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself says nothing at all.

5.531 So I do not write "f(a,b) . a = b" but "f(a,a)" (or "f(b,b)"). And not "f(a,b) . ~a = b" but "f(a,b)".

5.532 And similarly: not "(∃x,y).f(x,y) . x = y" but "(∃x).f(x,x)"; and not "(∃x,y).f(x,y) . ~x = y" but "(∃x,y).f(x,y)".

(So instead of Russell's "(∃x,y). f(x,y)": "(∃x,y).f(x,y) .∨. (∃x).f(x,x)".)

5.5321 So instead of "(x):fx ⊃ x = a" we write, for example, "(∃x).fx .⊃. fa : ~(∃x,y).fx . fy".

And the proposition "*only* one x satisfies f()" reads: "(∃x).fx : ~(∃x,y).fx . fy".

5.533 The sign of identity is therefore not an essential constituent of a concept-script.

5.534 And now we see that pseudo-propositions like "a = a", "a = b . b = c .⊃ a = c", "(x).x = x", "(∃x).x = a", etc. cannot be written down at all in a correct concept-script.

5.535 This also disposes of all the problems that were bound up with such pseudo-propositions.

All the problems that Russell's "Axiom of Infinity" brings with it can already be solved here.

What the Axiom of Infinity is meant to say would be expressed in language by there being infinitely many names with different meanings.

5.5351 There are certain cases in which one is tempted to use expressions of the form "a = a" or "p ⊃ p" and the like. This happens when one wants to talk about the prototype: proposition, thing, etc. Thus in "Principles of Mathematics" Russell rendered the nonsense "p is a proposition" in symbols by "p ⊃ p" and placed it as a hypothesis in front of certain propositions, so that their argument places could only be occupied by propositions.

(It is nonsense to place the hypothesis p ⊃ p in front of a proposition to ensure that it gets arguments of the right form, if only because the hypothesis, given a non-proposition as argument, becomes not false but nonsensical, and because the proposition itself becomes nonsensical when given the wrong kind of arguments, and so it protects itself against the wrong arguments just as well, or just as badly, as the senseless hypothesis attached for that purpose.)

5.5352 In the same way, people wanted to express "There are no *things*" by "~(∃x).x = x". But even if this were a proposition—would it not also be true if there "were things" but they were not identical with themselves?

5.54 In the general propositional form, a proposition occurs in a proposition only as a base of truth-operations.

5.541 At first sight it looks as if a proposition could also occur in another in a different way.

Particularly in certain propositional forms of psychology, such as "A believes that p is the case" or "A thinks p", etc.

For here it looks superficially as if the proposition p stood in some kind of relation to an object A.

(And in modern theory of knowledge (Russell, Moore, etc.) these propositions have indeed been understood in this way.)[3]

5.542 But it is clear that "A believes that p", "A thinks p", "A says p" are of the form "'p' says p": and here what is at issue is not a correlation of a fact with an object, but the correlation of facts by means of the correlation of their objects.

5.5421 This also shows that the soul—the subject, etc.—as it is conceived in the superficial psychology of today, is an absurdity.

For a composite soul would no longer be a soul.

5.5422 The correct explanation of the form of the proposition "A judges p" must show that it is impossible to judge a nonsense. (Russell's theory does not satisfy this condition.)

5.5423 To perceive a complex means to perceive that its constituents are related to one another in such-and-such a way.

This presumably also explains why the figure

[Figure: cube]

can be seen as a cube in two ways; and all similar phenomena. For we really do see two different facts.

(If I look first at the corners a and only glance at b, a appears in front; and vice versa.)

5.55 We must now answer a priori the question about all the possible forms of elementary propositions.

3 G. E. Moore (1873–1958), English philosopher and Russell's colleague at Cambridge.

An elementary proposition consists of names. But since we cannot give the number of names with different meanings, we cannot give the composition of the elementary proposition either.

5.551 Our fundamental principle is that every question that can be decided by logic at all must be decidable straight off.

(And if we get into a position where we have to answer such a problem by looking at the world, this shows that we are on a fundamentally wrong track.)

5.552 The "experience" we need in order to understand logic is not that something is so-and-so, but that something *is*: but that is precisely *not* an experience.

Logic is *prior to* every experience—that something is *so*.

It is prior to the How, not prior to the What.

5.5521 And if this were not so, how could we apply logic? One could say: if there were a logic even if there were no world, how then could there be a logic, given that there is a world?

5.553 Russell said there were simple relations between different numbers of things (individuals). But between which numbers? And how is that to be decided?—By experience?

(There is no privileged number.)

5.554 Giving any particular form would be completely arbitrary.

5.5541 It is supposed to be possible to say a priori whether, for example, I may find myself in a position where I have to signify something with the sign of a 27-place relation.

5.5542 But may we even ask such a thing at all? Can we set up a form of sign and not know whether anything could correspond to it?

Does the question make sense: what must *be* in order for something to be-the-case?

5.555 It is clear that we have a concept of the elementary proposition, apart from its particular logical form.

But where symbols can be constructed according to a system, it is the system that matters logically, and not the individual symbols.

And how could it be that in logic I should be dealing with forms that I can invent? What I must be dealing with is what makes it possible for me to invent them.

5.556 There cannot be a hierarchy of the forms of elementary propositions. Only what we construct ourselves can we foresee.

5.5561 Empirical reality is limited by the totality of objects. The limit shows itself again in the totality of elementary propositions.

The hierarchies are, and must be, independent of reality.

5.5562 If we know on purely logical grounds that there must be elementary propositions, then everyone who understands propositions in their unanalyzed form must know it.

5.5563 All the propositions of our everyday language are actually, just as they are, in perfect logical order.—That simplest thing we are supposed to give here is not a likeness of the truth but the full truth itself.

(Our problems are not abstract but perhaps the most concrete there are.)

5.557 The *application* of logic decides which elementary propositions there are.

What lies in the application, logic cannot anticipate.

This much is clear: logic must not clash with its application.

But logic must be in contact with its application.

So logic and its application must not overlap.

5.5571 If I cannot give the elementary propositions a priori, then trying to give them must lead to obvious nonsense.

5.6 *The limits of my language* mean the limits of my world.

5.61 Logic fills the world; the limits of the world are also its limits.

So we cannot say in logic: this and this exists in the world, that does not.

For that would apparently presuppose that we exclude certain possibilities, and this cannot be the case, since otherwise logic would have to go beyond the limits of the world: that is, if it could view these limits from the other side as well.

What we cannot think, we cannot think; so we also cannot *say* what we cannot think.

5.62 This remark provides the key to deciding the question of how far solipsism is a truth.

For what solipsism *means* is quite correct; only it cannot be *said*, but shows itself.

That the world is *my* world shows itself in the fact that the limits of *the* language (the language that I alone understand) mean the limits of *my* world.[4]

5.621 The world and life are one.

5.63 I am my world. (The microcosm.)

5.631 The thinking, presenting subject does not exist.

If I wrote a book called "The World as I Found It", I would also have to report on my body in it, and say which parts are subject to my will and which are not, etc. This is in fact a method of isolating the subject, or rather of showing that in an important sense there is no subject: for it alone could *not* be mentioned in that book.—

5.632 The subject does not belong to the world; rather, it is a limit of the world.

5.633 Where *in* the world is a metaphysical subject to be found?

You say that this is just like the case of the eye and the visual field. But you really do *not* see the eye.

And nothing *in the visual field* allows you to infer that it is seen by an eye.

5.6331 For the visual field does not have, say, a form like this:

Eye—[Figure: eye and visual field]

5.634 This is connected with the fact that no part of our experience is also a priori.

Everything we see could also be otherwise.

Everything we can describe at all could also be otherwise.

There is no a priori order of things.

5.64 Here we see that solipsism, strictly carried through, coincides with pure realism. The I of solipsism shrinks to an exten-

4 Solipsism: the view that only oneself and one's own experience can be known to exist.

sionless point, and what remains is the reality coordinated with it.

5.641 So there really is a sense in which philosophy can talk about the I non-psychologically.

The I enters philosophy through the fact that "the world is my world".

The philosophical I is not the human being, not the human body, or the human soul that psychology deals with, but the metaphysical subject, the limit—not a part—of the world.

Proposition 6

The general form of a truth-function

6 The general form of a truth-function is: [p̄, ξ̄, N(ξ̄)].

This is the general form of a proposition.

6.001 This says nothing other than that every proposition is a result of successive applications of the operation N'(ξ̄) to the elementary propositions.

6.002 If we are given the general form of how a proposition is constructed, then we are thereby also given the general form of how one proposition can be generated from another by means of an operation.

6.01 The general form of an operation Ω'(η̄) is therefore: [ξ̄, N(ξ̄)]' (η̄) (= [η̄, ξ̄, N(ξ̄)]).

This is the most general form of transition from one proposition to another.

6.02 And *this* is how we arrive at numbers. I define

x = Ω^0'x Def. and

Ω'Ω^{ν}'x = Ω^{ν+1}'x Def.

Following these rules for signs, then, we write the series

x, Ω'x, Ω'Ω'x, Ω'Ω'Ω'x, ...

as:

Ω^0'x, Ω^{0 + 1}'x, Ω^{0 + 1 + 1}'x, Ω^{0 + 1 + 1 + 1}'x, ...

So instead of "[x, ξ, Ω'ξ]" I write:

"[Ω^0'x, Ω^{ν}'x, Ω^{ν + 1}'x]".

And I define:

0 + 1 = 1 Def.

0 + 1 + 1 = 2 Def.

0 + 1 + 1 + 1 = 3 Def.

(and so on)

6.021 A number is the exponent of an operation.

6.022 The concept of number is nothing other than what all numbers have in common: the general form of number.

The concept of number is the variable number.

And the concept of numerical equality is the general form of all particular numerical equalities.

6.03 The general form of a whole number is: [0, ξ, ξ + 1].

6.031 The theory of classes is completely superfluous in mathematics.

This is connected with the fact that the generality we need in mathematics is not *accidental* generality.

6.1 The propositions of logic are tautologies.

6.11 The propositions of logic therefore say nothing. (They are the analytic propositions.)

6.111 Theories that make a proposition of logic appear to have content are always false. One might believe, for example, that the words "true" and "false" signify two properties among other properties; and then it would seem a remarkable fact that every proposition possesses one of these properties. That now seems anything but self-evident—no more self-evident than, say, the proposition "all roses are either yellow or red" would sound, even if it were true. Indeed, the former proposition now takes on the full character of a proposition of natural science, and this is the sure sign that it has been misconceived.

6.112 The correct explanation of the propositions of logic must give them a unique position among all propositions.

6.113 It is the special mark of logical propositions that one can recognize from the symbol alone that they are true; and this fact contains within it the whole philosophy of logic. And so it is also one of the most important facts that the truth or falsity of non-logical propositions can *not* be recognized from the proposition alone.

6.12 That the propositions of logic are tautologies *shows* the formal—logical—properties of language, of the world.

That their constituents, connected *in this way*, yield a tautology characterizes the logic of their constituents.

For propositions connected in a particular way to yield a tautology, they must have particular properties of structure. So the fact that, connected *in this way*, they yield a tautology shows that they possess these properties of structure.

6.1201 That, for example, the propositions "p" and "~p" in the combination "~(p . ~p)" yield a tautology shows that they contradict each other. That the propositions "p ⊃ q", "p" and "q", combined with one another in the form "(p ⊃ q) . (p) :⊃: (q)", yield a tautology shows that q follows from p and p ⊃ q. That "(x).fx :⊃: fa" is a tautology shows that fa follows from (x).fx. Etc., etc.

6.1202 It is clear that one could use contradictions instead of tautologies for the same purpose.

6.1203 To recognize a tautology as such, one can, in cases where no sign of generality occurs in the tautology, use the following visual method. Instead of "p", "q", "r", etc., I write "W p F", "W q F", "W r F", etc. I express the truth-combinations by means of brackets, for example:

[Figure]

and the correlation of the truth or falsity of the whole proposition with the truth-combinations of the truth-arguments by means of lines, in the following way:

[Figure]

So this sign, for example, would represent the proposition p ⊃ q. Now I want to examine, for example, the proposition ~(p . ~p) (the law of contradiction) to find out whether it is a tautology. In our notation the form "~ξ" is written

[Figure]

and the form "ξ . η" thus:

[Figure]

Hence the proposition ~(p . ~q) reads as follows:

[Figure]

If we substitute "p" for "q" here and examine how the outermost W and F are connected with the innermost, it turns out that the truth of the whole proposition is correlated with *all* the

truth-combinations of its argument, and its falsity with none of them.

6.121 The propositions of logic demonstrate the logical properties of propositions by combining them into propositions that say nothing.

This method could also be called a zero-method. In a logical proposition, propositions are brought into equilibrium with one another, and the state of equilibrium then indicates how these propositions must be logically constituted.[1]

6.122 It follows that we can also get by without logical propositions, since in a suitable notation we can recognize the formal properties of propositions simply by looking at those propositions.

6.1221 If, for example, two propositions "p" and "q" in the combination "p ⊃ q" yield a tautology, then it is clear that q follows from p.

That "q" follows from "p ⊃ q . p", for example, we see from these two propositions themselves; but we can also show it *in this way*: by combining them into "p ⊃ q . p :⊃: q" and then showing that this is a tautology.

6.1222 This throws light on the question why logical propositions cannot be confirmed by experience, any more than they can be refuted by experience. Not only must it be impossible for any possible experience to refute a proposition of logic; it must also be impossible for any such experience to confirm it.

6.1223 Now it becomes clear why people have often felt as though "logical truths" were ours to "*demand*": we can in fact demand them insofar as we can demand an adequate notation.

6.1224 It also becomes clear now why logic has been called the theory of forms and of inference.

6.123 It is clear: the laws of logic cannot themselves be subject to further laws of logic.

1 A zero-method (null method) in physics measures a quantity by balancing it against a known one until an instrument reads zero, as with a balance scale.

(There is not, as Russell thought, a special law of contradiction for each "type"; one is enough, since it is not applied to itself.)

6.1231 The mark of a logical proposition is *not* general validity.

To be general, after all, means only to happen to hold for all things. An ungeneralized proposition can be tautological just as well as a generalized one.

6.1232 Logical general validity could be called essential, in contrast to the accidental kind—that of the proposition "all human beings are mortal", for instance. Propositions like Russell's "Axiom of reducibility" are not logical propositions, and this explains our feeling that, if they are true, they could only be true by a lucky accident.[2]

6.1233 A world can be imagined in which the Axiom of reducibility does not hold. But it is clear that logic has nothing to do with the question whether our world really is like that or not.

6.124 The logical propositions describe the scaffolding of the world, or rather they represent it. They are not "about" anything. They presuppose that names have meaning and elementary propositions have sense; and this is their connection with the world. It is clear that it must indicate something about the world that certain combinations of symbols—which essentially have a definite character—are tautologies. This is the decisive point. We said that some things in the symbols we use are arbitrary and some are not. In logic only the latter express anything. But that means that in logic it is not *we* who express what we want with the help of signs; rather, in logic the nature of the naturally necessary signs itself speaks. If we know the logical syntax of any sign-language, then all the propositions of logic are already given.

6.125 It is possible—even on the old conception of logic—to give a description of all "true" logical propositions in advance.

6.1251 That is why there can *never* be surprises in logic.

2 An axiom of *Principia Mathematica,* introduced to make the theory of types workable for mathematics; Wittgenstein cites it in English.

6.126 Whether a proposition belongs to logic can be calculated by calculating the logical properties of the *symbol*.

And this is what we do when we "prove" a logical proposition. For without concerning ourselves with a sense or a meaning, we form the logical proposition from others by mere *rules for signs*.

The proof of logical propositions consists in our producing them from other logical propositions by the successive application of certain operations that always generate tautologies again from the original ones. (And indeed only tautologies *follow* from a tautology.)

Naturally this way of showing that its propositions are tautologies is entirely inessential to logic—if only because the propositions from which the proof starts must show, without proof, that they are tautologies.

6.1261 In logic, process and result are equivalent. (Hence no surprise.)

6.1262 Proof in logic is merely a mechanical aid for recognizing a tautology more easily where it is complicated.

6.1263 It would indeed be too remarkable if one could prove a proposition with sense *logically* from others, and a logical proposition *as well*. It is clear from the outset that the logical proof of a proposition with sense and proof *in* logic must be two entirely different things.

6.1264 A proposition with sense states something, and its proof shows that it is so; in logic every proposition is the form of a proof.

Every proposition of logic is a modus ponens represented in signs. (And modus ponens cannot be expressed by a proposition.)[3]

6.1265 Logic can always be conceived in such a way that every proposition is its own proof.

6.127 All the propositions of logic are of equal standing; there is no essential division among them into primitive laws and derived propositions.

3 *Modus ponens* (Latin, "the mode that affirms"): the rule of inference that from p and p ⊃ q one may infer q.

Every tautology itself shows that it is a tautology.

6.1271 It is clear that the number of "primitive laws of logic" is arbitrary, since one could derive logic from a single primitive law simply by forming, for example, the logical product of Frege's primitive laws. (Frege would perhaps say that this primitive law was then no longer immediately self-evident. But it is remarkable that a thinker as exact as Frege should have appealed to the degree of self-evidence as the criterion of a logical proposition.)

6.13 Logic is not a body of doctrine but a mirror image of the world.

Logic is transcendental.

6.2 Mathematics is a logical method.

The propositions of mathematics are equations, and therefore pseudo-propositions.

6.21 A proposition of mathematics does not express a thought.

6.211 In life it is never the mathematical proposition that we need; rather, we use mathematical propositions *only* in order to infer, from propositions that do not belong to mathematics, other propositions that likewise do not belong to mathematics.

(In philosophy the question "What do we actually use that word, that proposition, for?" leads again and again to valuable insights.)

6.22 The logic of the world, which the propositions of logic show in tautologies, mathematics shows in equations.

6.23 If two expressions are connected by the sign of equality, this means that they can be substituted for each other. But whether this is the case must show itself in the two expressions themselves.

It characterizes the logical form of two expressions that they can be substituted for each other.

6.231 It is a property of affirmation that it can be conceived as double negation.

It is a property of "1 + 1 + 1 + 1" that it can be conceived as "(1 + 1) + (1 + 1)".

6.232 Frege says that the two expressions have the same meaning but a different sense.

But what is essential about an equation is that it is not needed in order to show that the two expressions connected by the sign of equality have the same meaning, since this can be seen from the two expressions themselves.

6.2321 And that the propositions of mathematics can be proved means nothing other than that their correctness can be seen without our having to compare what they express with the facts to check whether it is correct.

6.2322 The identity of meaning of two expressions cannot be *asserted*. For in order to be able to assert something about their meaning, I must know their meaning; and in knowing their meaning, I know whether they mean the same thing or different things.

6.2323 An equation marks only the point of view from which I consider the two expressions: namely, the point of view of their equality of meaning.

6.233 To the question whether intuition is needed to solve mathematical problems, the answer must be that here it is language itself that supplies the necessary intuition.[4]

6.2331 It is precisely the process of *calculating* that brings about this intuition.

A calculation is not an experiment.

6.234 Mathematics is a method of logic.

6.2341 The essence of the mathematical method is working with equations. For it is on this method that the fact rests that every proposition of mathematics must be intelligible by itself.

6.24 The method by which mathematics arrives at its equations is the method of substitution.

For equations express the substitutability of two expressions, and we proceed from a number of equations to new equations by replacing expressions with others in accordance with the equations.

4 Intuition (*Anschauung*): Kant's term for the immediate, sensible grasp of space and time on which, in his view, mathematics rests.

6.241 Thus the proof of the proposition $2 \times 2 = 4$ runs:

$(\Omega^{\nu})^{\mu}\text{'}x = \Omega^{\nu \times \mu}\text{'}x$ Def.

$\Omega^{2 \times 2}\text{'}x = (\Omega^2)^{2}\text{'}x = (\Omega^2)^{1 + 1}\text{'}x = \Omega^{2}\text{'}\Omega^{2}\text{'}x$

$= \Omega^{1 + 1}\text{'}\Omega^{1 + 1}\text{'}x = (\Omega\text{'}\Omega)\text{'}(\Omega\text{'}\Omega)\text{'}\,x$

$= \Omega\text{'}\Omega\text{'}\Omega\text{'}\Omega\text{'}x = \Omega^{1 + 1 + 1 + 1}\text{'}x = \Omega^{4}\text{'}x.$

6.3 The investigation of logic means the investigation of *all conformity to law*. And outside logic everything is accident.

6.31 The so-called law of induction cannot in any case be a law of logic, since it is obviously a proposition with sense.—And for that reason it cannot be an a priori law either.

6.32 The law of causality is not a law but the form of a law.

6.321 "Law of causality"—that is a generic name. And just as in mechanics there are, let us say, minimum laws—such as the law of least action—so in physics there are laws of causality, laws of the causal form.[5]

6.3211 Indeed, people had an inkling that there must be *a* "law of least action" before they knew exactly what it said. (Here, as always, what is certain a priori turns out to be something purely logical.)

6.33 We do not *believe* a priori in a law of conservation; rather, we *know* a priori the possibility of a logical form.

6.34 All those propositions—such as the principle of sufficient reason, the principle of continuity in nature, the principle of least effort in nature, etc., etc.—all of these are a priori insights into the possible forms that can be given to the propositions of science.

6.341 Newtonian mechanics, for example, brings the description of the world into a unified form. Let us imagine a white surface with irregular black spots on it. We now say: whatever kind of picture results from this, I can always come as close as I like to a description of it by covering the surface with a suitably fine network of squares and then saying of each square that it is white or black. In this way I will have brought the description of the surface into a unified form. This form is arbitrary, since I

5 The principle of least action states that a physical system moves along the path for which a certain quantity, the action, is smallest.

could have used a net with triangular or hexagonal meshes with equal success. It may be that the description using a triangular net would have been simpler; that is, that we could describe the surface more accurately with a coarser triangular net than with a finer square one (or vice versa), and so on. Different systems for describing the world correspond to the different nets. Mechanics determines one form of description of the world by saying: all propositions in the description of the world must be obtained in a given way from a number of given propositions—the axioms of mechanics. In this way it supplies the bricks for putting up the edifice of science, and it says: whatever building you want to erect, you must somehow assemble it with these bricks, and with these alone.

(Just as with the number system one must be able to write down any number whatever, so with the system of mechanics one must be able to write down any proposition of physics whatever.)

6.342 And now we can see the relative positions of logic and mechanics. (The net could also consist of figures of different kinds, for instance of triangles and hexagons.) That a picture like the one just mentioned can be described by a net of a given form says *nothing* about the picture. (For this holds of every picture of this kind.) What does characterize the picture, though, is that it can be *completely* described by a particular net of a *particular* fineness.

In the same way, it says nothing about the world that it can be described by Newtonian mechanics; but it does say something that it can be described by it in *just the way* it in fact is. It also says something about the world that it can be described more simply by one mechanics than by another.

6.343 Mechanics is an attempt to construct all the *true* propositions we need for describing the world according to a single plan.

6.3431 Through the whole logical apparatus, the laws of physics still speak of the objects of the world.

6.3432 We must not forget that the description of the world by mechanics is always entirely general. It never speaks, for example, of *particular* material points, but always only of *any* material points whatever.

6.35 Although the spots in our picture are geometrical figures, geometry can obviously say nothing at all about their actual shape and position. But the net is *purely* geometrical; all its properties can be given a priori.

Laws like the principle of sufficient reason, etc., are about the net, not about what the net describes.

6.36 If there were a law of causality, it might run: "There are laws of nature".

But of course that cannot be said: it shows itself.

6.361 In Hertz's terminology one could say: only connections that are *subject to law* are *thinkable*.

6.3611 We cannot compare any process with the "passage of time"—there is no such thing—but only with another process (such as the working of a chronometer).

Hence the course of time can be described only by relying on some other process.

Something exactly analogous holds for space. When people say, for example, that neither of two events (which exclude each other) can occur, because there is *no cause* why the one should occur rather than the other, what is really at issue is that one cannot even describe *one* of the two events unless there is some asymmetry. And *if* there *is* such an asymmetry, we can regard it as the *cause* of the one event's occurring and the other's not occurring.

6.36111 Kant's problem about the right and the left hand, which cannot be made to coincide, already exists in the plane—indeed in one-dimensional space, where the two congruent figures a and b likewise cannot be made to coincide without being moved



a b

out of this space. Right and left hands are in fact completely congruent. And the fact that they cannot be made to coincide has nothing to do with that.

A right-hand glove could be put on the left hand if it could be turned around in four-dimensional space.[6]

6.362 What can be described can also happen; and what the law of causality is supposed to exclude cannot even be described.

6.363 The process of induction consists in our adopting the *simplest* law that can be brought into agreement with our experiences.

6.3631 This process, however, has no logical justification, only a psychological one.

It is clear that there is no reason to believe that the simplest case will in fact occur.

6.36311 That the sun will rise tomorrow is a hypothesis; and that means: we do not *know* whether it will rise.

6.37 There is no compulsion by which one thing would have to happen because something else has happened. There is only *logical* necessity.

6.371 The whole modern view of the world rests on the illusion that the so-called laws of nature are the explanations of natural phenomena.

6.372 So people stop at the laws of nature as at something inviolable, as people in earlier times stopped at God and fate.

And both are right, and wrong. But the ancients are clearer in that they acknowledge a clear endpoint, while the new system tries to make it look as if *everything* has been explained.

6.373 The world is independent of my will.

6.374 Even if everything we wish for were to happen, this would still only be, so to speak, a favor granted by fate; for there is no *logical* connection between will and world that would guarantee it, and the supposed physical connection is not something we could in turn will.

6 Kant discusses such "incongruent counterparts" in the *Prolegomena* (1783), §13: mirror-image objects that are alike in every inner respect yet cannot occupy the same space.

6.375 Just as there is only *logical* necessity, so too there is only *logical* impossibility.

6.3751 For two colors, for example, to be at one place in the visual field at the same time is impossible—logically impossible, in fact, since it is ruled out by the logical structure of color.

Let us consider how this contradiction is represented in physics. Roughly like this: a particle cannot have two velocities at the same time; that is, it cannot be in two places at the same time; that is, particles in different places at one and the same time cannot be identical.

(It is clear that the logical product of two elementary propositions can be neither a tautology nor a contradiction. The statement that a point in the visual field has two different colors at the same time is a contradiction.)

6.4 All propositions are of equal value.

6.41 The sense of the world must lie outside it. In the world everything is as it is, and everything happens as it happens; *in* it there is no value—and if there were, it would have no value.

If there is a value that does have value, it must lie outside everything that happens and everything that is so. For everything that happens and everything that is so is accidental.

What makes it non-accidental cannot lie *in* the world, for then this would again be accidental.

It must lie outside the world.

6.42 That is why there can be no propositions of ethics either.

Propositions cannot express anything higher.

6.421 It is clear that ethics cannot be put into words.

Ethics is transcendental.

(Ethics and aesthetics are one.)

6.422 The first thought when an ethical law of the form "you shall . . . ." is laid down is: And what if I don't do it? But it is clear that ethics has nothing to do with punishment and reward in the ordinary sense. So this question about the *consequences* of an action must be irrelevant.—At least these consequences must not be events. For there must be something right about the way that question is posed. There must indeed be a kind of ethical

reward and ethical punishment, but these must lie in the action itself.

(And it is also clear that the reward must be something pleasant and the punishment something unpleasant.)

6.423 We cannot speak of the will as the bearer of the ethical.

And the will as a phenomenon is of interest only to psychology.

6.43 If good or evil willing changes the world, it can change only the limits of the world, not the facts—not what can be expressed by language.

In short, the world must then, through this, become a different world altogether. It must, so to speak, shrink or grow as a whole.

The world of the happy is a different world from that of the unhappy.

6.431 Just as, at death, too, the world does not change but ceases.

6.4311 Death is not an event in life. We do not live through death.

If by eternity we understand not infinite duration in time but timelessness, then whoever lives in the present lives eternally.

Our life is endless in just the way that our visual field is boundless.

6.4312 The temporal immortality of the human soul—that is, its eternal survival after death—is not only in no way guaranteed; above all, this assumption does not do at all what people have always wanted it to do. Is some riddle solved by my surviving forever? Is this eternal life not, then, just as much of a riddle as our present one? The solution of the riddle of life in space and time lies *outside* space and time.

(It is not problems of natural science, after all, that have to be solved.)

6.432 *How* the world is, is a matter of complete indifference for what is higher. God does not reveal himself *in* the world.

6.4321 The facts all belong only to the problem, not to the solution.

6.44 The mystical is not *how* the world is, but *that* it is.

6.45 The contemplation of the world sub specie aeterni [under the aspect of eternity] is its contemplation as a—limited—whole.

The feeling of the world as a limited whole is the mystical.[7]

6.5 When an answer cannot be put into words, neither can the question.

The riddle does not exist.

If a question can be asked at all, then it *can* also be answered.

6.51 Skepticism is *not* irrefutable, but obviously nonsensical, when it tries to doubt where no question can be asked.

For doubt can exist only where a question exists; a question only where an answer exists; and an answer only where something *can* be *said*.

6.52 We feel that even when all *possible* scientific questions have been answered, the problems of our life have not yet been touched at all. Of course, there is then no question left; and precisely this is the answer.

6.521 The solution of the problem of life is seen in the vanishing of this problem.

(Is this not the reason why people to whom the sense of life became clear after long doubt could not then say what that sense consisted in?)

6.522 There is indeed what cannot be put into words. This *shows* itself; it is the mystical.

6.53 The correct method in philosophy would really be this: to say nothing except what can be said—that is, propositions of natural science, and so something that has nothing to do with philosophy—and then, whenever someone else wanted to say something metaphysical, to demonstrate to him that he had given no meaning to certain signs in his propositions. This method would be unsatisfying to the other person—he would not have the feeling that we were teaching him philosophy—but *it* would be the only strictly correct one.

6.54 My propositions elucidate in this way: whoever understands me eventually recognizes them as nonsensical, once he

has climbed through them—on them—beyond them. (He must, so to speak, throw away the ladder after he has climbed up it.)

He must surmount these propositions; then he sees the world rightly.

7 The Latin phrase echoes Spinoza, who speaks in his *Ethics* (1677) of knowing things *sub specie aeternitatis*.

Proposition 7

What we cannot speak about we must pass over in silence

[7] What we cannot speak about we must pass over in silence.